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Open strings in the plane wave background II: Superalgebras and Spectra

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ABSTRACT

In hep-th/0211011 we started a systematic investigation of open strings in the plane wave background. In this paper we continue the analysis by discussing the superalgebras of conserved charges, the spectra of open strings, and the spectra of DBI fluctuations around D-brane embeddings. We also derive the gluing conditions for corresponding boundary states and analyze their symmetries. All results are consistent with each other, and confirm the existence of additional supersymmetries as previously discussed. We further show that for every symmetry current one can construct a (countably) infinite number of related currents that contain more worldsheet derivatives, and discuss non-local symmetries.

Contents

1 Introduction

The maximally supersymmetric plane wave background of type IIB string theory [1] presents a useful model for studying string theory on backgrounds that contain RR-fluxes. Such backgrounds play a prominent role in various gauge theory/gravity dualities. The plane wave background is particularly interesting because it is related to the $AdS_5 \times S^5$ background via a Penrose limit [2]. This implies a correspondence between the plane wave and gauge theory and such a relation has been proposed in [3]. This model is also distinguished by the fact that the worldsheet is free in the lightcone gauge [4]. Closed string theory on the plane wave background was discussed in [4, 5].

In our previous paper [6] we started a systematic investigation of open strings in the plane wave background. For related earlier work see [7, 8, 9, 10, 11] (a more complete list of references can be found in [6]). We studied possible boundary conditions for open strings in the lightcone gauge and discussed the global symmetries of the worldsheet action. The branes fall into equivalences classes depending on whether they related by the action of a target space isometry. Given a set of boundary conditions, some of the target space symmetries are broken. One may then act by the broken generators on the boundary conditions to generate a new set of boundary conditions. These branes are physically equivalent since they are related by an isometric reparametrization of the target spacetime. Notice that static and time-dependent branes can be part of the same equivalence class. This is so because some of the target space isometries depend on x^+ which is identified with the worldsheet time τ in the lightcone gauge, so the action of such a broken target space symmetry on time-independent boundary conditions (static D-brane) can produce time-dependent boundary conditions (time-dependent brane).

Recall that in the plane wave geometry the transverse coordinates to the lightcone are divided into two sets of four. The D-branes are divided into D_- and D_+ branes [6][1]. The former are branes of the type $(+,-,m,m\pm 2)$, where the notation indicates that the brane wraps the lightcone directions and m $(m\pm 2)$ of the worldvolume coordinates wrap the first (second) set of the transverse coordinates. The remaining branes $(+,-,m,n)$ are the D_+ branes. The division of the branes into these two sets originates from differences in the fermionic boundary conditions.

[1]The same classification was also introduced in [12] that appeared shortly after [6]. The D_- branes are called "class I" and the D_+ branes "class II" in [12]. The $\pm$ in our notation is motivated by the fact that the matrix that is associated with the boundary conditions for the fermions "squares" to -1 for the D_- branes, and $+1$ for the D_+ branes, see (2.6).

One might expect that all symmetries of the open string are symmetries of the closed string which are compatible with the open string boundary conditions. We showed in [6], however, that some of the naively broken closed string symmetries are restored using certain worldsheet symmetries. In all such cases the violation of the closed string symmetry depended solely on quantities that are determined by the boundary conditions, and the violating terms could be adjusted to zero to changing the boundary condition. For example, D_- branes located at a constant transverse position $x_0^{r'}$ appear to break all dynamical supersymmetries, and the violating terms vanish when the transverse position is set to zero, $x_0^{r'} = 0$.

In all such cases, however, we find that there are other worldsheet "symmetries" that are also violated by exactly the same amount as the closed string symmetries under discussion. It follows that the combination of the closed string and worldsheet symmetry is a good symmetry of the open string action. In the example of the D_- branes, we find that even though the worldsheet action breaks all dynamical supersymmetries when the brane is located away from the origin, it preserves eight fermionic symmetries that are linear combinations of eight dynamical supersymmetries and worldsheet symmetries. Furthermore, the algebra of the new fermionic symmetries is the standard one, i.e. their anticommutator gives the lightcone Hamiltonian (plus other conserved charges), so they may be called "dynamical" supersymmetries.

The worldsheet symmetries originate from the fact that the action is quadratic in the fields. This implies that the transformations that shift each worldsheet field by a parameter that satisfies the worldsheet field equations leave the Lagrangian invariant up to total derivatives. Transformations of this type that do *not* respect the boundary conditions are the ones used in order to restore seemingly broken target space symmetries. The transformations that do respect the boundary condition give rise to new worldsheet symmetries. Expanding the parameter in a basis we find that for both the closed and the open string there is a countably infinite number of such worldsheet symmetries. The corresponding currents evaluated on-shell are equal to oscillators.

The worldsheet symmetries we discuss here are very familiar in the context of conformal field theories, but to our knowledge they have not been used before in the way we use them. As is discussed in textbooks, given a holomorphic current $J(z)$, $\bar{\partial} J = 0$, there are an infinite number of other conserved currents, namely $J_n = z^n J$. For example, for a free boson $X(z)$, the worldsheet action is invariant under $X \to X + \epsilon(z)$. Expanding $\epsilon(z)$ in a basis we get a countably infinite global worldsheet symmetries generated by $J_n = z^n \partial X$, where n is an integer (we consider worldsheets without boundaries in this example). The corresponding charges evaluated on-shell are proportional to oscillators. In more general models, such as the ones in [13], where the worldsheet theory is integrable, the worldsheet symmetries of our discussion should be related to the infinite number of conservation laws associated with the integrability of the model.

In this paper we continue the analysis started in [6]. First we compute on-shell the symmetry charges, and we use the resulting expressions to determine the superalgebra they satisfy. The spectrum of the open strings should organize into representations of this algebra, and we show that this is indeed the case. In particular, the supersymmetries that are restored by worldsheet symmetries act on the spectrum properly.

As discussed, when the brane is located away from the origin, some of the symmetries are restored by the use of worldsheet symmetries. In all such cases, the corresponding charge expressed in terms of oscillators is exactly the same (up to certain c-number contributions for some charges) as the corresponding charge for a brane located at the origin. This immediately implies that the superalgebras are also the same.

In the case of symmetry-related branes, the conserved charges of all branes in the same equivalence class are related to each other in a way that follows from the relation of the corresponding boundary conditions. We verify that the corresponding symmetry algebras are the same once the relations between the generators are taken into account.

We should comment here that although identifying the symmetries is evidently important in any theory, the superalgebra plays a particularly central role in lightcone Green-Schwarz string theory. As is well-known, a convenient way to compute three point and higher functions is to use lightcone string field theory. A basic principle in the development of lightcone string field theory is to add interaction terms to dynamical charges in such a way as to preserve the superalgebra [14, 15, 16]. Thus the algebras we discuss in detail here would be important in calculating open string amplitudes, following the discussions of closed string field theory in the plane wave initiated in [17].

The multiplet generated by the action of zero modes on the vacuum (which in the flat space limit is massless) should appear in the DBI action as the multiplet of small fluctuations around the corresponding D-brane embedding. We verify that this is indeed the case for the bosonic fluctuations about D_+1 and $D3$ embeddings. In the latter case we also consider a time-dependent embedding and again find exact agreement.

In [6] we found, as is reviewed above, that some symmetries that are naively broken are actually replaced by new symmetries. Basically all configurations that according to the probe analysis [8] preserve at least 1/4 of the closed string supersymmetries are found to preserve 16 supercharges; the extra supercharges are due to the new symmetries. The D_+ branes that the probe analysis showed to be non-supersymmetric, are now found to preserve 8 new kinematical supersymmetries. Inspection of the spectrum of these branes shows that the massive string states contain an equal number of bosons and fermions at each energy level. The same degeneracy appears in the spectrum of other branes, and in that case it is explained by the existence of the dynamical supersymmetry: the states are related to each other by the action of the dynamical supercharges. This leads us to investigate the existence of yet another fermionic symmetry that would act as a dynamical supersymmetry in this case.

We do find additional conserved fermionic currents but the construction in all cases is effectively non-local. One may start with a local conserved fermionic current that is closely related to the dynamical supersymmetry current of the $D1$ brane. The addition of the corresponding charge to the algebra, however, induces an infinite number of additional charges. Each of these charges is associated with a local current, and the currents are related to each other by the addition of worldsheet derivatives. Thus, the closure of the algebra requires the use of currents with an infinite number of worldsheet derivatives and the construction is effectively non-local. Alternatively, one may start from a charge written in terms of modes that acts properly on the spectrum and ask whether there is a local current that generates it. We show that there is no local current that is associated with the charge and the corresponding symmetry transformations are non-local. It thus seems likely that the degeneracy of the massive string states will be lifted by loop effects.

An additional outcome of this analysis is that we find, for both open and closed strings, that for each worldsheet current there are (countably) infinite associated currents. These are obtained from the original one by judious additions of worldsheet derivatives to the local expression for the current. This is reminiscent of the higher spin symmetries in higher dimensional free field theories. It would be interesting to investigate whether there are any relations between the higher spin symmetries of free $N = 4, d = 4$ SYM theory and the new symmetries just mentioned.

The discussion of the spectrum and the DBI fluctuations confirms that the extra symmetries are symmetries of the spectrum. This still leaves open the possibility that these symmetries are broken by the interactions. A set of interactions that are straightforward to describe are the ones between closed and open strings and static interactions between a pair of D-branes. Such interactions are determined using boundary states. Boundary states in the plane wave background were constructed in [7, 10, 12], following the flat space analysis in [18]. Our discussion follows the discussion of boundary states in the RNS formalism [19, 20, 21]. We derive the gluing conditions by considering appropriate boundary conditions for the worldsheet field and imposing the latter as operator relations. In the cases in common, our results agree with the ones in [7, 10, 12]. Once the boundary state is thus defined, one may investigate how many symmetries it preserves. We find that for all branes that preserve sixteen supersymmetries (some of which may be the new supersymmetries that use worldsheet symmetries), there is a corresponding boundary state that preserves 16 supercharges. This is in particular the case for D_- branes located away from the origin.

This paper is organized as follows. In the next section we review the quantization of the open string and the corresponding mode expansions from [6]. In section 3 we evaluate on-shell the conserved charges derived in [6] and compute the corresponding superalgebras. Section 4 addresses the issue of the existence of additional fermionic symmetries for the D_+ branes that do not possess any dynamical supersymmetry. We also discuss in this section higher derivative currents and non-local symmetries. In section 5 we discuss the brane spectra and in section 6 we compute the spectrum of small fluctuations around certain D-brane embeddings derived in [8]. Section 7 contains the discussion of boundary states and their symmetries. Finally there are three appendices where we review relevant material from the corresponding analysis of closed strings and give our conventions.

2 Review of quantization

In this section we briefly review the results obtained in [6], see also [9, 11, 12]. In what follows we will consider open strings propagating in the maximally supersymmetric plane wave background [1], henceforth called the plane wave, with Brinkmann metric

$$ds^2 = 2dx^+dx^- + \sum_{I=1}^{8}(dx^I dx^I - \mu^2(x^I)^2(dx^+)^2), \tag{2.1}$$

and RR flux

$$F_{+1234} = F_{+5678} = 4\mu. \tag{2.2}$$

Our starting point is the worldsheet action in the lightcone and conformal gauge

$$\begin{aligned} S &= T\int d^2\sigma \left(p^+\partial_\tau x^- + \tfrac{1}{2}((\partial_\tau x^I)^2 - (\partial_\sigma x^I)^2 - m^2(x^I)^2)\right. \\ &\quad \left. + i(\theta^1\bar{\gamma}^-\partial_+\theta^1 + \theta^2\bar{\gamma}^-\partial_-\theta^2 - 2m\theta^1\bar{\gamma}^-\Pi\theta^2)\right). \end{aligned} \tag{2.3}$$

Here $\partial_\pm = \partial_\tau \pm \partial_\sigma$, $m = \mu p^+$, $\Pi = \gamma^{1234}$, and T is the tension of the string ($T = 2\pi\alpha'$ for the closed string and $T = \pi\alpha'$ for the open string; we set $\alpha' = 1$ throughout this paper). Note also that we rescale the fermions by a factor of $\sqrt{p^+}$ here relative to [6].

The closed string mode expansion and the canonical commutation relations were worked out in [4] and are reviewed in appendix C. In [6] we analyzed possible boundary conditions and the canonical quantization of the open strings. In all cases the fermions satisfy the boundary condition

$$\theta^1| = \Omega\theta^2|, \tag{2.4}$$

where $A|$ indicates evaluation at the boundary (i.e. $\sigma = 0$ and $\sigma = \pi$). The orthogonal matrix Ω is equal to the product of the transverse to the brane gamma matrices[2],

$$\Omega = \prod_{r'=p}^{8} \gamma^{r'}. \tag{2.5}$$

The branes are divided into two cases: the D_- and D_+ branes,

$$D_- : \quad \Omega\Pi\Omega\Pi = -1, \qquad D_+ : \quad \Omega\Pi\Omega\Pi = 1. \tag{2.6}$$

The D_- branes are of the type $(+, -, m, m \pm 2)$ and the D_+ branes are the remaining $(+, -, m, n)$ cases.

The mode expansions of the bosonic coordinates for both D_+ and D_- branes are given by

$$x^r(\sigma,\tau) = x_0^r \cos(m\tau) + m^{-1} p_0^r \sin(m\tau) + i \sum_{n\neq 0} \omega_n^{-1} \alpha_n^r e^{-i\omega_n \tau} \cos(n\sigma); \tag{2.7}$$

$$x^{r'}(\sigma,\tau) = x_0^{r'}(\sigma,\tau) + \sum_{n\neq 0} \omega_n^{-1} \alpha_n^{r'} e^{-i\omega_n \tau} \sin(n\sigma), \tag{2.8}$$

where

$$\omega_n = \mathrm{sgn}(n)\sqrt{n^2 + m^2}. \tag{2.9}$$

The exact form of the zero mode part, $x_0^{r'}(\sigma,\tau)$, depends on the boundary conditions under consideration. For a static brane located at $x_0^{r'}$ it is

$$x_0^{r'}(\sigma,\tau) = \frac{x_0^{r'}}{e^{m\pi} + 1}(e^{m\sigma} + e^{m(\pi-\sigma)}), \tag{2.10}$$

[2] Our index conventions are as follows: we use indices $r, s, t = 1, .., (p-1)$, from the end of the latin alphabet to denote coordinates with Neumann boundary conditions, and primed indices of the same letters, $r', s', t' = p, .., 8$, to denote coordinates with Dirichlet boundary conditions. Latin indices, $i, j = 1, .., 4$, and $i', j' = 5, .., 8$, from the middle of the alphabet are used for the two sets of the transverse coordinates that transform among themselves under the $SO(4)$ and the $SO(4)'$, respectively.

whilst for the symmetry related branes it is

$$x_0^{r'}(\sigma,\tau) = a^{r'}\cos(m\tau) + b^{r'}\sin(m\tau), \tag{2.11}$$

where both $a^{r'}$ and $b^{r'}$ are constant vectors in the Dirichlet directions.

The equal time commutator of the oscillators is given by

$$[a_n^I, a_l^J] = \mathrm{sgn(n)}\delta_{n+l}\delta^{IJ}, \qquad [\bar{a}_0^r, a_0^s] = \delta^{rs}, \tag{2.12}$$

where one defines

$$a_0^r = \frac{1}{\sqrt{2m}}(p_0^r + imx_0^r), \quad \bar{a}_0^r = \frac{1}{\sqrt{2m}}(p_0^r - imx_0^r), \quad a_n^I = \sqrt{\frac{1}{|\omega_n|}}\alpha_n^I. \tag{2.13}$$

2.1 D_- branes

The fermion mode expansions are

$$\begin{aligned} \theta^1 &= \theta_0\cos(m\tau) + \tilde{\theta}_0\sin(m\tau) + \sum_{n\neq 0} c_n\left(id_n\Pi\theta_n\phi_n + \tilde{\theta}_n\tilde{\phi}_n\right); \\ \theta^2 &= \Pi\tilde{\theta}_0\cos(m\tau) - \Pi\theta_0\sin(m\tau) + \sum_{n\neq 0} c_n\left(-id_n\Pi\tilde{\theta}_n\tilde{\phi}_n + \theta_n\phi_n\right), \end{aligned} \tag{2.14}$$

where the expansion functions are given in (B.14), n is an integer and

$$d_n = \frac{1}{m}(\omega_n - n), \qquad c_n = \frac{1}{\sqrt{1+d_n^2}}. \tag{2.15}$$

Furthermore,

$$\tilde{\theta}_0 = -\Omega\Pi\theta_0; \quad \tilde{\theta}_n = \Omega\theta_n. \tag{2.16}$$

The anticommutators of the fermions are given by

$$\{\theta_0^\alpha, \theta_0^\beta\} = \frac{1}{8}(\gamma^+)^{\alpha\beta}, \quad \{\theta_n^\alpha, \theta_m^\beta\} = \frac{1}{8}(\gamma^+)^{\alpha\beta}\delta_{n+m,0}. \tag{2.17}$$

We will find it useful to use pure Dirichlet and pure Neumann combinations of these fermions such that

$$\theta_D(\sigma,\tau) = (\theta^1 - \Omega\theta^2)(\sigma,\tau) = 2\sum_n c_n e^{-i\omega_n\tau}(d_n\Pi + i\Omega)\sin(n\sigma)\theta_n; \tag{2.18}$$

$$\begin{aligned} \theta_N(\sigma,\tau) &= (\theta^1 + \Omega\theta^2)(\sigma,\tau) \\ &= 2(\theta_0\cos(m\tau) - \Omega\Pi\theta_0\sin(m\tau)) + 2\sum_{n\neq 0} c_n e^{-i\omega_n\tau}(id_n\Pi + \Omega)\cos(n\sigma)\theta_n, \end{aligned} \tag{2.19}$$

which are manifestly both expansions in orthogonal functions [6].

2.2 D_+ branes

The fermionic mode expansion in this case is given by

$$\begin{aligned} \theta^1 &= \theta_0^+ e^{m\sigma} + \theta_0^- e^{-m\sigma} + \sum_{n\neq 0} c_n \left(i d_n \Pi \theta_n \phi_n + \tilde{\theta}_n \tilde{\phi}_n \right); \\ \theta^2 &= \Pi\theta_0^+ e^{m\sigma} - \Pi\theta_0^- e^{-m\sigma} + \sum_{n\neq 0} c_n \left(-i d_n \Pi \tilde{\theta}_n \tilde{\phi}_n + \theta_n \phi_n \right), \end{aligned} \tag{2.20}$$

where one uses the definitions in (B.14) and (2.15). The boundary condition enforces

$$\begin{aligned} \theta_0^{\pm} &= \pm\Omega\Pi\theta_0^{\pm}; \\ \tilde{\theta}_n &= c_n^2(\Omega(1-d_n^2) - 2id_n\Pi)\theta_n. \end{aligned} \tag{2.21}$$

The anticommutators of the fermionic oscillators are given by

$$\begin{aligned} \{\theta_n^\alpha, \theta_m^\beta\} &= \frac{1}{8}(\gamma^+)^{\alpha\beta}\delta_{n+m,0}; \\ \{\mathcal{P}_+\theta_0^+, \mathcal{P}_+\theta_0^+\} &= \frac{\pi m}{4(e^{2\pi m}-1)}\mathcal{P}_+\gamma^+; \\ \{\mathcal{P}_-\theta_0^-, \mathcal{P}_-\theta_0^-\} &= \frac{\pi m e^{2\pi m}}{4(e^{2\pi m}-1)}\mathcal{P}_-\gamma^+. \end{aligned} \tag{2.22}$$

where $\mathcal{P}_\pm = \frac{1}{2}(1 \pm \Omega\Pi)$. It is convenient to rescale these zero modes as

$$\hat{\theta}_0^+ = \sqrt{\frac{(e^{2\pi m}-1)}{2\pi m}}\theta_0^+; \quad \hat{\theta}_0^- = \sqrt{\frac{(1-e^{-2\pi m})}{2\pi m}}\theta_0^-, \tag{2.23}$$

in terms of which the commutation relations are

$$\{\hat{\theta}_0, \hat{\theta}_0\} = \frac{1}{8}\gamma^+. \tag{2.24}$$

where we denote $\hat{\theta}_0 = (\hat{\theta}_0^+ + \hat{\theta}_0^-)$. Again it is useful to define pure Dirichlet and Neumann combinations, such that

$$\theta_D(\sigma,\tau) = (\theta^1 - \Omega\theta^2)(\sigma,\tau) = 2\sum_n c_n e^{-i\omega_n\tau}(d_n\Pi + i\Omega)\sin(n\sigma)\theta_n, \tag{2.25}$$

$$\theta_N(\sigma,\tau) = (\theta^1 + \Omega\theta^2)(\sigma,\tau) - m\int^\sigma d\sigma'(\Pi\theta^2 + \Omega\Pi\theta^1)(\sigma',\tau). \tag{2.26}$$

The latter is Neumann because using the field equations and the boundary conditions its sigma derivative vanishes on the boundary:

$$\partial_\sigma\theta_N(\sigma,\tau) = 2\sum_n c_n\omega_n e^{-i\omega_n\tau}(id_n\Pi - \Omega)\sin(n\sigma)\theta_n. \tag{2.27}$$

3 Superalgebras

In this section we explicitly evaluate the conserved charges identified in [6] in terms of modes, and compute the superalgebras. Throughout the conserved charge G is given in terms of the τ component of the symmetry current $\mathcal{G}^\tau$ by

$$G = T\int_0^l d\sigma \mathcal{G}^\tau, \tag{3.1}$$

where l is the length of the string, 2π for a closed string and π for an open string. We review in appendix B the τ components of the symmetry currents discussed in detail in [6].

3.1 Closed Strings

Recall that the target space superalgebra is generated by the momenta $P^\pm$ and P^I and the rotation generators J^{IJ}, J^{+I} and the (complex) kinematical supersymmetries Q^+ and dynamical supersymmetries Q^-. The closed string (super)charges generate the superalgebra of the plane wave background, which is given in appendix B. The realization of this algebra in terms of closed string modes was worked out in [5] and is reviewed in appendix C. As we have seen in [6] the kinematical charges, P^I, J^{+I} and Q^{+1}, Q^{+2}, are members of an infinite family of symmetries and thus one may extend the superalgebra to include these charges as well. The charges $P^{\mathcal{I}I}_m, Q^{\mathcal{I}I}_n, \mathcal{I} = 1, 2$, evaluated on-shell using the currents given in appendix B and the closed string mode expansions reviewed in appendix C, are given by

$$\begin{aligned}
&P^{1I}_n = 2\alpha^{1I}_n, \qquad P^{2I}_n = 2\alpha^{2I}_n, \qquad n \neq 0, \\
&Q^1_n = 2\bar{\gamma}^-\theta^1_n, \qquad Q^2_n = 2\bar{\gamma}^-\theta^2_n, \qquad n \neq 0.
\end{aligned} \tag{3.2}$$

Using these we can compute the extension of the superalgebra. Since the new charges are proportional to oscillators the superalgebra is extended by the oscillator (anti) commutation relations (C.5). With the definitions given above,

$$[P^{\mathcal{I}I}_m, P^{\mathcal{J}J}_n] = 2\omega_m\delta_{m+n,0}\delta^{IJ}\delta^{\mathcal{I}\mathcal{J}}, \qquad \{Q^{\mathcal{I}}_m, Q^{\mathcal{J}}_n\} = 2\bar{\gamma}^-\delta^{\mathcal{I}\mathcal{J}}\delta_{m+n,0}. \tag{3.3}$$

Furthermore, $P^{\mathcal{I}K}_n$ transform as vectors and $Q^{\mathcal{I}}_n$ as spinors under $SO(4)\times SO(4)'$ and they are supersymmetric partners with respect to the dynamical supersymmetry,

$$\begin{aligned}
&[J^{IJ}, P^{\mathcal{I}K}_n] = i(\delta^{IK}P^{\mathcal{I}J}_n - \delta^{JK}P^{\mathcal{I}J}_n), \qquad [J^{IJ}, Q^{\mathcal{I}}_n] = -\frac{i}{2}Q^{\mathcal{I}}_n\gamma^{IJ}, \\
&[Q^{-\mathcal{I}}, P^{\mathcal{J}I}_n] = -\delta^{\mathcal{I}\mathcal{J}}\omega_n c_n\gamma^{I+}Q^{\mathcal{J}}_n - \epsilon^{\mathcal{I}\mathcal{J}}\frac{im}{2c_n}\gamma^{I+}\Pi Q^{\mathcal{J}}_n, \\
&\{Q^{-\mathcal{I}}, Q^{\mathcal{J}}_n\} = \delta^{\mathcal{I}\mathcal{J}}c_n P^{\mathcal{J}I}_n\bar{\gamma}^I\gamma^+\bar{\gamma}^- - \epsilon^{\mathcal{I}\mathcal{J}}\frac{im}{2\omega_n c_n}P^{2I}_n\bar{\gamma}^I\Pi\gamma^+\bar{\gamma}^-.
\end{aligned}$$

Finally the commutation with the lightcone Hamiltonian ($P^- = -H$) are given by

$$[P^-, P_n^{\mathcal{I}I}] = \omega_n P_n^{\mathcal{I}I}, \qquad [P^-, Q_n^{\mathcal{I}}] = \omega_n Q_n^{\mathcal{I}}. \tag{3.4}$$

This is an infinite extension of the target space superalgebra. It would be interesting to understand the form of the extended algebra in covariant gauges.

Note that the worldsheet charges are spectrum generating. The commutation relation (3.4) implies that states related by the action of the worldsheet charges have different lightcone energy.

3.2 D_- branes

We determine in this section the charges identified in [6] as being conserved in terms of the modes. This computation provides a nice check of the identification of the conserved currents in [6]: the corresponding charges when evaluated on-shell must be time independent. In this subsection we discuss branes with static (ordinary Dirichlet) boundary conditions as given in (2.10); we consider the symmetry-related branes with time dependent Dirichlet boundary conditions (2.11) in section 3.4.

3.2.1 Conserved charges

The momentum currents are given by the same expressions (B.2) as for the closed string. We thus get for conserved momenta

$$P^+ = p^+, \qquad P^r = \sqrt{p^+} p_0^r, \tag{3.5}$$

where by the arguments of [6] there is no conserved charge $P^{r'}$. The Hamiltonian density is given in (B.5) and evaluating on-shell we get

$$\begin{aligned}
H &= \Delta H + E_0 + E_N; \\
\Delta H &= \frac{m(e^{m\pi} - 1)}{\pi(e^{m\pi} + 1)} \sum_{r'=p}^{8} (x_0^{r'})^2; \\
E_0 &= m \left(\sum_{r=1}^{p-1} a_0^r \bar{a}_0^r - 2i\theta_0 \bar{\gamma}^- \Omega \Pi \theta_0 + \tfrac{1}{2}(p-1) \right); \\
E_N &= \sum_{n>0} \left(\omega_n a_{-n}^I a_n^I + 4\omega_n \theta_{-n} \bar{\gamma}^- \theta_n \right).
\end{aligned} \tag{3.6}$$

There is a zero point energy as a result of normal ordering harmonic oscillator zero modes. For the non-zero modes, the normal ordering constants cancel out between bosonic and fermionic oscillators.

There is in addition a shift in the energy ΔH as a result of moving the brane away from the origin. One can understand this physically as follows. Suppose we consider a classical static string ending on a brane which is displaced from the origin. This corresponds to considering only the time independent modes in our bosonic solutions, so that

$$x^r = 0; \qquad x^{r'} = (e^{m\pi}+1)^{-1} x_0^{r'} (e^{m\sigma} + e^{m(\pi-\sigma)}). \tag{3.7}$$

This solution describes a string whose endpoints are at $x_0^{r'}$ and whose midpoint is at

$$x^{r'} = x_0^{r'} (\cosh(\tfrac{1}{2} m\pi))^{-1} < x_0^{r'}. \tag{3.8}$$

Thus the string bends towards $x^{r'} = 0$ and has finite proper length, in contrast to the $m = 0$ (flat space limit) when $x^{r'}$ is constant and so the proper length of the string is zero.

One may think that because of the extra energy ΔH in the Hamiltonian in the case of the D-brane located away from the origin the string would want to move to the origin to minimize its energy. The superalgebra, however, given in (3.16) but with $P^- \to P^- + \Delta H$, implies a BPS bound and the latter is saturated by the states with energy ΔH. Furthermore, the analysis of small fluctuations in section 6 also shows that the brane does not tend to move towards the origin; the brane rather oscillates around the constant transverse position. The extra energy ΔH only affects the frequency of oscillations. Notice that these considerations are valid for a single brane located at a non-zero constant position, and do not imply that there is no force between *two* branes one located at the origin and another at some non-zero constant position – to analyze this issue one should study open strings with one end on one brane and one end on the other. This can be done straightforwardly using the methods developed in [6] and here, but we shall not pursue it further in this paper. We also refer to [10, 12] for discussions of $p - p'$ strings.

The rotation currents are given in (B.3). Evaluating on-shell we get

$$\begin{aligned} J^{+r} &= -\sqrt{p^+} x_0^r, \\ J^{rs} &= -i(a_0^r \bar{a}_0^s - a_0^s \bar{a}_0^r + \theta_0 \gamma^{-rs} \theta_0) - i \sum_{n>0} \left(a_{-n}^r a_n^s - a_{-n}^s a_n^r + 2\theta_{-n} \gamma^{-rs} \theta_n \right), \\ J^{r's'} &= -i\theta_0 \gamma^{-r's'} \theta_0 - i \sum_{n>0} \left(a_{-n}^{r'} a_n^{s'} - a_{-n}^{s'} a_n^{r'} + 2\theta_{-n} \gamma^{-r's'} \theta_n \right). \end{aligned} \tag{3.9}$$

Notice that, as we showed in [6], the current $\mathcal{J}^{r's'}$ in (B.3) is conserved only when $x_0^{r'} = x_0^{s'} = 0$. When there are non-zero Dirichlet zero modes, the conserved current $\hat{\mathcal{J}}^{r's'}$ is a combination of $\mathcal{J}^{r's'}$ with a worldsheet current [6]. It is actually equal to $\mathcal{J}^{r's'}$ but with $x^{r'} \to (x^{r'} - x_0^{r'})$. It follows that the corresponding charge $\hat{J}^{r's'}$ is on-shell equal to $J^{r's'}$.

Note that $J^{rs'} = 0$ because the corresponding symmetry does not respect the boundary conditions.

In [6] we showed that these open string boundary conditions lead to preserved kinematical charges q^+ where $q^+ = \frac{1}{2}(\bar{\Omega}Q^{+1} - Q^{+2})$ as well as preserved dynamical supersymmetries q^- where $q^- = \frac{1}{2}(Q^{-1} + \bar{\Omega}Q^{-2})$ when the brane is at the origin in transverse position. These Noether charges take the form

$$\begin{aligned} q^+ &= \frac{1}{\pi}\int_0^\pi d\sigma \sqrt{p^+}\bar{\gamma}^-(\cos(m\tau) + \sin(m\tau)\Omega\Pi)\theta_N; \qquad (3.10) \\ q^- &= \frac{1}{\pi}\int_0^\pi d\sigma \left(\sum_{r=1}^{p-1}(\partial_\tau x^r \bar{\gamma}^r \theta_N - \partial_\sigma x^r \bar{\gamma}^r \theta_D + m x^r \bar{\gamma}^r \Omega\Pi\theta_N) \right. \\ & \left. \sum_{r'=p}^{8}(\partial_\tau x^{r'} \bar{\gamma}^{r'} \theta_D - \partial_\sigma x^{r'} \bar{\gamma}^{r'} \theta_N - m x^{r'} \bar{\gamma}^{r'} \Omega\Pi\theta_D) \right), \end{aligned}$$

where here we use the pure Dirichlet and Neumann combinations $\theta_D = (\theta^1 - \Omega\theta^2)$ and $\theta_N = (\theta^1 + \Omega\theta^2)$ introduced in (2.18). Since the currents are now expressed in terms of mutually orthogonal functions it is straightforward to explicitly demonstrate that they are time independent and to evaluate the charges in terms of modes as

$$q^+ = 2\sqrt{p^+}\bar{\gamma}^-\theta_0, \qquad (3.11)$$

$$\begin{aligned} q^- &= \left(2p_0^r \bar{\gamma}^r \theta_0 - 2m x_0^r \bar{\gamma}^r \Pi\Omega^t \theta_0\right) \\ &+ \sum_{n>0}\left(2\sqrt{\omega_n}c_n(a_n^r \bar{\gamma}^r - a_n^{r'}\bar{\gamma}^{r'})\Omega\theta_{-n} - \frac{im}{\sqrt{\omega_n}c_n}(a_n^I \bar{\gamma}^I)\Pi\theta_{-n}\right) \\ &+ \sum_{n>0}\left(2\sqrt{\omega_n}c_n(a_{-n}^r \bar{\gamma}^r - a_{-n}^{r'}\bar{\gamma}^{r'})\Omega\theta_{n} + \frac{im}{\sqrt{\omega_n}c_n}(a_{-n}^I \bar{\gamma}^I)\Pi\theta_{n}\right). \qquad (3.12) \end{aligned}$$

As discussed in [6], the charge in (3.10) is only a conserved supercharge when there are no Dirichlet zero modes but when there are zero modes of the form (2.10), the conserved charge is instead $\hat{q}^-$, the combination of q^- with the worldsheet symmetry, which is

$$\begin{aligned} \hat{q}^- &= \frac{1}{\pi}\int_0^\pi d\sigma \left(\sum_{r=1}^{p-1}(\partial_\tau x^r \bar{\gamma}^r \theta_N - \partial_\sigma x^r \bar{\gamma}^r \theta_D + m x^r \bar{\gamma}^r \Omega\Pi\theta_N) \right. \qquad (3.13) \\ & \left. \sum_{r'=p}^{8}(\partial_\tau (x^{r'} - x_0^{r'}) \bar{\gamma}^{r'} \theta_D - \partial_\sigma (x^{r'} - x_0^{r'}) \bar{\gamma}^{r'} \theta_N - m (x^{r'} - x_0^{r'}) \bar{\gamma}^{r'} \Omega\Pi\theta_D) \right). \end{aligned}$$

This charge when realized in terms of modes reproduces precisely q^-.

The conserved worldsheet symmetries are given by

$$P_n^I = a_n^I; \quad Q_n = 2\bar{\gamma}^-\theta_n; \quad n \neq 0, \qquad (3.14)$$

where in the second expression the open string symmetry Q_n is related to the closed string symmetries reviewed in appendix B as $Q_n = \frac{1}{2}(Q_n^2 + \bar{\Omega}^t Q_n^1)$.

3.2.2 D_- superalgebra

One can now use the commutation relations to obtain the superalgebra that the (super)charges satisfy. As we just discussed the conserved charges in terms of oscillators for static branes located at and away from the origin are the same (with the exception of the Hamiltonian which is shifted by the c-number ΔH), even though the relations of the currents to the corresponding currents of the closed string are different. In the latter case the conserved currents are linear combinations of closed string currents with certain worldsheet currents but in the former case they are not. It follows that the algebras we now discuss apply equally to both cases. The only difference is that one should replace P^- by $\hat{P}^- = P^- + \Delta H$, where ΔH is given in (3.6). As discussed in section 4.2.1 of [6], ΔH is associated with a certain worldsheet symmetry, see (4.31)-(4.32) in [6].

Part of the superalgebra is given by a restriction of the closed string algebra reviewed in appendix B. We give here the non-trivial relations:

$$
\begin{aligned}
\left[P^r, q^-\right] &= -\tfrac{1}{2} i m q^+(\Pi\Omega^t \gamma^{+r}), \qquad \left[P^-, q^+\right] = imq^+(\Omega\Pi), \\
\{q^+, q^+\} &= P^+ \bar{\gamma}^-, \\
\{q^+, q^-\} &= \tfrac{1}{2}((\bar{\gamma}^- \gamma^+ \bar{\gamma}^r) P^r + m(\bar{\gamma}^- \gamma^+ \bar{\gamma}^r \Pi\Omega^t) J^{+r}), \\
\{q^-, q^-\} &= \bar{\gamma}^+ P^- - \tfrac{1}{2} m(\gamma^{+rs} \Pi\Omega^t) J^{rs} - \tfrac{1}{2} m(\gamma^{+r's'} \Pi\Omega^t) J^{r's'}.
\end{aligned}
\tag{3.15}
$$

In the last line, one uses Π when $(r, r' \subset i)$ but one should replace Π by Π' when $(r, r' \subset i')$. The derivation of these results is standard, but somewhat involved when it comes to the fermion bilinears as one needs to use the Fierz rearrangement formulas given in appendix A to bring the right hand side to the form quoted. Since the anticommutator $\{q^-, q^-\}$ is a symmetric matrix in the spinor indices, only symmetric products of gamma matrices can appear. One can indeed check that only γ^+ and $\gamma^{+m_1 m_2 m_3 m_4}$ appear. For later use we record the terms proportional to γ^+ for various D_- branes (we give here the expressions for the $(+, -, m+2, m)$ branes, but the $(+, -, m, m+2)$ cases are exactly analogous),

$$
\begin{aligned}
&(+, -, 2, 0): \qquad && \{q^-, q^-\} = \bar{\gamma}^+(P^- + mJ^{34}) + \cdots \\
&(+, -, 3, 1): \qquad && \{q^-, q^-\} = \bar{\gamma}^+ P^- + \cdots \\
&(+, -, 4, 2): \qquad && \{q^-, q^-\} = \bar{\gamma}^+(P^- + mJ^{56}) + \cdots
\end{aligned}
\tag{3.16}
$$

where $J^{34} \subset J^{ij}$ is the generator of rotations along the transverse to the $D3$ brane directions and $J^{56} \subset J^{i'j'}$ is the generator of rotations along worldvolume coordinates in the $D7$ case. The dots indicate terms proportional to $\gamma^{+m_1m_2m_3m_4}$, whose exact form can be read-off from (3.15).

The extension of the superalgebra by the worldsheet charges can be worked out exactly as in the case of closed strings. The (anti-)commutation relations among themselves are, up to normalization factors, the basic (anti)-commutation relations of the modes (2.12)-(2.17). The commutation relations of P_n^I and Q_n with J^{IJ} reflect their transformation properties under rotations. The remaining commutators are

$$\begin{aligned}
[P^-, P_n^I] &= \omega_n P_n^I, \quad [P^-, Q_n] = \omega_n Q_n, \\
[q^-, P_n^r] &= -\frac{1}{4\sqrt{|\omega_n|}c_n}\left(2\omega_n c_n^2 \bar{\gamma}^r \Omega + im\bar{\gamma}^r \Pi\right)\gamma^+ Q_n, \\
\left[q^-, P_n^{r'}\right] &= \frac{1}{4\sqrt{|\omega_n|}c_n}\left(2\omega_n c_n^2 \bar{\gamma}^{r'} \Omega - im\bar{\gamma}^{r'} \Pi\right)\gamma^+ Q_n, \\
\{q^-, Q_n\} &= \left(\frac{1}{2}\sqrt{|\omega_n|}c_n(P_n^r\bar{\gamma}^r - P_n^{r'}\bar{\gamma}^{r'})\Omega - \frac{im\omega_n}{4\sqrt{|\omega_n|}c_n}P_n^I\bar{\gamma}^I\Pi\right)\gamma^+\bar{\gamma}^-.
\end{aligned} \tag{3.17}$$

This extended algebra is a subalgebra of the extended closed string algebra. The embedding is obtained by taking the open string charges $(P^+, P^r, J^{+r}, J^{rs}, J^{r's'})$ to be the same as the closed string ones. P^- is also the same when the brane is located at the origin; otherwise we need to remove the ΔH part of the open string P^-, as discussed. The remaining charges are related by

$$\begin{aligned}
&q^+ = \tfrac{1}{2}(-Q^{+2} + \bar{\Omega}Q^{+1}); \quad q^- = \tfrac{1}{2}(Q^{-1} + \bar{\Omega}Q^{-2}); \quad Q_n = \tfrac{1}{2}(Q_n^2 + \bar{\Omega}^t Q_n^1); \\
&P_n^r = \frac{1}{2\sqrt{|\omega_n|}}(P_n^{1r} + P_n^{2r}); \quad P_n^{r'} = \frac{1}{2\sqrt{|\omega_n|}}(-P_n^{1r'} + P_n^{2r'}).
\end{aligned} \tag{3.18}$$

The latter expressions follow from forming cosine and sine combinations respectively of the functions $(\phi_n, \tilde{\phi}_n)$ appearing in the closed string mode expansions. With these identifications one can show that the D_- brane algebra is indeed a subalgebra of the extended closed algebra.

In the closed string algebra the commutation relations between supercharges involve charges such as $P^{r'}$ which are broken by the brane. It is thus instructive to show explicitly how these broken charges drop out of the open string commutation relations. Using the relations $\{Q^{\pm\mathcal{I}}, Q^{\pm\mathcal{J}}\}$ following from the closed string algebra given in appendix B we find that

$$\begin{aligned}
\{q^+, q^-\} &= \tfrac{1}{4}\{\bar{\Omega}Q^{+1} - Q^{+2}, Q^{-1} + \bar{\Omega}Q^{-2}\}; \\
&= \tfrac{1}{4}\bar{\gamma}^-\gamma^+\left((\bar{\gamma}^I + \bar{\Omega}\bar{\gamma}^I\Omega^t)P^I + m(\bar{\gamma}^I\Pi\Omega^t - \bar{\Omega}\bar{\gamma}^I\Pi)J^{+I}\right).
\end{aligned} \tag{3.19}$$

Recalling that $\{\Omega, \gamma^{r'}\} = 0 = [\Omega, \gamma^r]$, the charges in the Dirichlet directions drop out, as indeed they must since they are explicitly broken by the D-brane, leaving

$$\{q^+, q^-\} = \tfrac{1}{2}(\bar{\gamma}^+\gamma^-\bar{\gamma}^r)P^r + m(\bar{\gamma}^+\gamma^-\bar{\gamma}^r\Pi\Omega^t)J^{+r}, \tag{3.20}$$

in agreement with our open string result (3.15). Similarly

$$\begin{aligned}
\{q^-, q^-\} &= \tfrac{1}{4}\{Q^{-1} + \bar{\Omega}Q^{-2}, Q^{-1} + \bar{\Omega}Q^{-2}\}; \qquad (3.21)\\
&= \bar{\gamma}^+P^- + \tfrac{1}{4}m(\bar{\Omega}\gamma^{+ij}\Pi - \gamma^{+ij}\Pi\Omega^t)J^{ij} + \tfrac{1}{4}m(\bar{\Omega}\gamma^{+i'j'}\Pi - \gamma^{+i'j'}\Pi\Omega^t)J^{i'j'}.
\end{aligned}$$

Terms involving the explicitly broken charges $J^{rr'}$ drop out because $\gamma^{rr'}$ anticommutes with Ω, and thus the anticommutator reproduces that in (3.15).

3.3 D_+ branes

We now discuss the D_+ branes, the notation referring to branes for which the fermion boundary condition is $\theta^1| = \Omega\theta^2|$ where $(\Omega\Pi)^2 = 1$.

3.3.1 Conserved charges

Substituting the mode expansions into the conserved charges we find that the momenta and angular momenta are

$$\begin{aligned}
P^+ &= p^+, \quad P^r = \sqrt{p^+}p_0^r, \quad J^{+r} = -\sqrt{p^+}x_0^r, \qquad (3.22)\\
J^{rs} &= -i(a_0^r\bar{a}_0^s - a_0^s\bar{a}_0^r + \hat{\theta}_0\gamma^{-rs}\hat{\theta}_0) - i\sum_{n>0}(a_{-n}^r a_n^s - a_{-n}^s a_n^r + 2\theta_{-n}\gamma^{-rs}\theta_n),\\
J^{r's'} &= -i\hat{\theta}_0\gamma^{-r's'}\hat{\theta}_0 - i\sum_{n>0}(a_{-n}^{r'}a_n^{s'} - a_{-n}^{s'}a_n^{r'} + 2\theta_{-n}\gamma^{-r's'}\theta_n).
\end{aligned}$$

Again the rotation charges in the Dirichlet directions are applicable only if the brane is located at the origin in the relevant transverse coordinates. Away from the origin, the conserved charge is instead $\hat{J}^{r's'}$ which takes exactly the same form when evaluated in terms of oscillators. The Hamiltonian is

$$\begin{aligned}
H &= \Delta H + E_0 + E_N;\\
\Delta H &= \frac{m}{\pi}\left(\frac{(e^{m\pi}-1)}{(e^{m\pi}+1)}\right)\sum_{r'=p}^{8}(x_0^{r'})^2; \qquad (3.23)\\
E_0 &= m\left(\sum_{r=1}^{p-1}a_0^r\bar{a}_0^r + \tfrac{1}{2}(p-1)\right);\\
E_N &= \sum_{n>0}(\omega_n a_{-n}^I a_n^I + 4\omega_n\theta_{-n}\bar{\gamma}^-\theta_n).
\end{aligned}$$

Notice particularly that the fermion zero modes do not contribute to the E_0 Hamiltonian in this case.

As discussed in [6] only in the case of the D1-brane are any of the closed string supercharges preserved. In this case the dynamical supercharges preserved are $q^- = \frac{1}{2}(Q^{-1} + \bar{\Omega}Q^{-2})$ such that

$$\begin{aligned} q^- &= \frac{1}{\pi}\int_0^\pi d\sigma \sum_{r'} \Big(\partial_\tau x^{r'} \bar{\gamma}^{r'} \theta_D - \partial_\sigma x^{r'} \bar{\gamma}^{r'} (\theta^1 + \Omega\theta^2) - m x^{r'} \bar{\gamma}^{r'} (\Pi\theta^2 + \Omega\Pi\theta^1)\Big) \\ &= \frac{1}{\pi}\int_0^\pi d\sigma \sum_{r'} \Big(\partial_\tau x^{r'} \bar{\gamma}^{r'} \theta_D + x^{r'} \bar{\gamma}^{r'} \partial_\sigma \theta_N\Big) - \frac{1}{\pi}[x^{r'} \bar{\gamma}^{r'} (\theta^1 + \Omega\theta^2)]_0^\pi, \end{aligned} \tag{3.24}$$

where we now use the Dirichlet and Neumann combinations given in (2.25) and (2.27). Explicitly evaluating this charge we find

$$\begin{aligned} q^- &= -2\sqrt{(2m/\pi)\tanh \tfrac{1}{2}m\pi}\, x_0^{r'} \bar{\gamma}^{r'} \Omega\Pi\hat{\theta}_0 \\ &\quad - \sum_{n>0} 2c_n\sqrt{\omega_n} a_n^{r'} \bar{\gamma}^{r'} (\Omega + i d_n \Pi)\theta_{-n} - \sum_{n>0} 2c_n\sqrt{\omega_n} a_{-n}^{r'} \bar{\gamma}^{r'} (\Omega - i d_n \Pi)\theta_n. \end{aligned} \tag{3.25}$$

All the D_+ branes also have the exceptional kinematical charges (B.17) which are realized in terms of the fermionic zero modes as

$$\hat{Q}^+ = 2\sqrt{p^+}\bar{\gamma}^-\hat{\theta}_0. \tag{3.26}$$

The remaining worldsheet charges are exactly the same in terms of modes as for the D_- branes, namely

$$P_n^I = a_n^I, \qquad Q_n = \bar{\gamma}^-\theta_n, \qquad n \neq 0, \tag{3.27}$$

where here the appropriate combination of closed string charges is $Q_n = \frac{1}{2}(Q_n^2 + c_n^2(\bar{\Omega}^t(1 - d_n^2) + 2i d_n \bar{\Pi})Q_n^1)$.

3.3.2 D_+ superalgebra

Let us first discuss the superalgebra for the D1-brane. The $J^{r's'}$ generators satisfy the standard commutation relations with themselves and q^- (and, as in previous cases, for the D1-brane shifted from the origin we have to replace $J^{r's'}$ by $\hat{J}^{r's'}$). The anticommutator of the dynamical supercharges is

$$\{q^-, q^-\} = \bar{\gamma}^+ P^-. \tag{3.28}$$

This result depends crucially on the fact that we are dealing with a D1 brane. In processing the fermion bilinear terms one needs to perform a Fierz rearrangement. The relevant formula is given in (A.12) and in general leads to terms that depend on $\gamma^{\mu_1 \cdot\cdot \mu_5}$. In the case of D_-

branes these terms correspond to the J^{rs} and $J^{r's'}$ terms in the anticommutation relation. In the current case, these terms vanish when $p = 1$ due to a gamma matrix identity. Furthermore, the recombination of the bosonic and fermionic contributions to yield P^- is also sensitive to the fact that $p = 1$.

The anti-commutators that involve kinematical supercharges are

$$\begin{aligned}
\{\hat{Q}^+, \hat{Q}^+\} &= \bar{\gamma}^- P^+, \\
\{q^-, \hat{Q}^+\} &= -\sqrt{(m/2\pi)\tanh\tfrac{1}{2}m\pi}(\sqrt{p^+}x_0^{r'})\bar{\gamma}^{r'}\Pi'\gamma^+\bar{\gamma}^-, \\
\left[H, \hat{Q}^+\right] &= 0.
\end{aligned} \tag{3.29}$$

Notice in particular that these kinematical supercharges commute with the Hamiltonian.

The extension of the algebra by the worldsheet charges, apart from the obvious relations, is

$$\begin{aligned}
[P^-, P_n^{r'}] &= \omega_n P_n^{r'}, \qquad [P^-, Q_n] = \omega_n Q_n, \\
\left[q^-, P_n^{r'}\right] &= \mathrm{sgn}(n)\tfrac{1}{2}\gamma^{r'+}c_n\sqrt{|\omega_n|}(\Omega - id_n\Pi)Q_n, \\
\{q^-, Q_n\} &= -\tfrac{1}{2}c_n\sqrt{|\omega_n|}P_n^{r'}\bar{\gamma}^+\gamma^-\bar{\gamma}^{r'}(\Omega + id_n\Pi).
\end{aligned} \tag{3.30}$$

For the other D_+ branes the spacetime part of the algebra includes only bosonic generators, P^-, P^r, J^{+s}, J^{st}, and the exceptional kinematical supercharges $\hat{Q}^+$. Their commutation relations are the expected ones. The extension of the algebra by the worldsheet charges P_n^I and Q_n is similar to all other cases we discussed, and their commutation relations are also the expected ones.

3.4 Symmetry-related D-branes

We now give the conserved charges and algebra of the symmetry related branes, for which the Dirichlet boundary conditions are of the generalized time dependent type (2.11). Consider first the bosonic charges. The conserved charges of the brane $(P^+, P^r, J^{+r}, J^{rs}, P_n^I)$ are unaffected. Explicitly evaluating the remaining conserved charges identified in [6] we find

$$\begin{aligned}
\hat{P}^- &= P^- + \mu\sqrt{p^+}\sum_{r'}(b^{r'}P^{r'} - ma^{r'}J^{+r'}) = P_s^- + \tfrac{1}{2}m^2(a^2+b^2); \\
\hat{J}^{r's'} &= J^{r's'} - \frac{1}{\sqrt{p^+}}(a^{r'}P^{s'} + a^{s'}P^{r'} + mb^{s'}J^{+r'} - mb^{r'}J^{+s'}) \\
&= J_s^{r's'} + m(a^{r'}b^{s'} - a^{s'}b^{r'});
\end{aligned} \tag{3.31}$$

where the expression of the charges in terms of fields is given in appendix B and P_s^- and $J_s^{r's'}$ denote the charges for the static branes, as given in (3.6) and (3.9), and we use the shorthand notation $a^2 = \sum_{r'} a^{r'}a^{r'}$.

Now consider the preserved supercharges. For the type D_- branes, the charges (q^+, Q_n) are unaffected by the time dependent boundary conditions and the conserved dynamical supercharge is

$$\hat{q}^- = \tfrac{1}{2}\left(Q^{-1} + \bar{\Omega}Q^{-2} + \tfrac{1}{2}\mu\sqrt{p^+}\sum_{r'}(b^{r'}\gamma^{r'+} - a^{r'}\bar{\gamma}^{r'}\Omega\Pi\gamma^+)(Q^{+2} + \bar{\Omega}Q^{+1})\right) = q_s^-, \quad (3.32)$$

where again q_s^- is the static brane charge. To prove this the following expression is useful:

$$\tfrac{1}{2}(Q^{+2} + \bar{\Omega}Q^{+1}) = -\sqrt{p^+}\bar{\gamma}^- \int_0^\pi d\sigma(\cos(m\tau) - \sin(m\tau)\Omega\Pi)\theta_D. \quad (3.33)$$

Similarly for the D_+ D1-brane the charges $(\hat{Q}^+, Q_n)$ are unaffected and the conserved dynamical supercharge is

$$\begin{aligned}\hat{q}^- &= \tfrac{1}{2}(Q^{-1} + \bar{\Omega}Q^{-2}) \quad (3.34)\\ &+\tfrac{1}{4}\mu\sqrt{p^+}\sum_{r'}\left((b^{r'}\gamma^{r'+} - a^{r'}\bar{\gamma}^{r'}\Omega\Pi\gamma^+)Q^{+2} + (b^{r'}\gamma^{r'+}\bar{\Omega} + a^{r'}\gamma^{r'+}\bar{\Pi})Q^{+1}\right),\end{aligned}$$

which again evaluates to give precisely the static charge q_s^-. For the other D_+ branes the preserved kinematical supercharges are also unaffected.

Thus since the conserved charges of these branes have the same mode expansions as the conserved charges of the static brane, except for $\hat{P}^-$ and $\hat{J}^{r's'}$ which differ only by c-numbers, the algebra is also the same as that for the static branes, except for terms involving these charges. The extended superalgebra for the symmetry-related branes hence reproduces (3.15) and (3.17), with charges G replaced by $\hat{G}$ and

$$\hat{P}^- \to \hat{P}^- - \tfrac{1}{2}m^2(a^2 + b^2); \qquad \hat{J}^{r's'} \to \hat{J}^{r's'} - ma^{r'}b^{s'} + ma^{s'}b^{r'}. \quad (3.35)$$

That is, we absorb these overall c-number shifts into the definitions of $\hat{G}$.

Although the open string algebras for these branes are precisely the same as for the static branes, the embeddings of the symmetry-related brane algebras into the closed string algebra differ from that for the static branes given in (3.18). The essential difference is in the combination of closed string supercharges which are preserved: for the D_- branes the open string dynamical charges $\hat{q}^-$ are now related to the closed string charges $Q^\pm$ as in (3.32).

Evaluating the anticommutator of $\hat{q}^-$ by directly substituting into the closed string superalgebra we find that

$$\{\hat{q}^-, \hat{q}^-\} = \bar{\gamma}^+(P^- + \mu\sqrt{p^+}\sum_{r'}(b^{r'}P^{r'} - ma^{r'}J^{+r'}) - \tfrac{1}{2}m^2(a^2+b^2)) - \tfrac{1}{2}m(\gamma^{+rs}\Pi\Omega^t)J^{rs}$$

$$-\tfrac{1}{2}m(\gamma^{+r's'}\Pi\Omega^t)\left(J^{r's'} - \frac{1}{\sqrt{p^+}}(a^{r'}P^{s'} + a^{s'}P^{r'} + m(b^{s'}J^{+r'} - b^{r'}J^{+s'}))\right.$$
$$\left. -m(a^{r'}b^{s'} - a^{s'}b^{r'})\right),$$

where appropriate summations over all Neumann and Dirichlet directions are implied. This agrees with the result above: the charges appearing on the right hand side are precisely the open string charges $\hat{G}$ including c-number shifts. One can also show that

$$\{q^+, q^+\} = P^+\bar{\gamma}^-; \quad \{q^+, \hat{q}^-\} = \tfrac{1}{2}(\bar{\gamma}^-\gamma^+\bar{\gamma}^r P^r - m\bar{\gamma}^-\gamma^+\bar{\gamma}^r\Pi\Omega^t J^{+r}), \tag{3.36}$$

i.e. the same as for the static D_- branes, with $q^- \to \hat{q}^-$. This also agrees with what we found above using the explicit form of the charges in terms of modes.

4 Are the remaining D_+-branes 1/2-supersymmetric?

In [6] we argued that there is no combination of closed string dynamical supercharges that preserves the boundary conditions for any D_+ brane except the D1 brane[3]: the symmetries are violated by terms involving the Neumann coordinates. We also showed in [6] that one cannot use worldsheet symmetries linear in the oscillators to restore this symmetry.

Following the logic of [6], in this section we will explore whether there is any symmetry of the D_+ branes corresponding to a dynamical supercharge which does not descend from closed string symmetries. We first note that the charge given in (3.24) for the $D1$-brane is conserved and preserves the boundary conditions for all D_+ branes. For reasons that will become apparent later in this section we shall call this charge q_0^-:

$$\begin{aligned} q_0^- &= \frac{1}{\pi}\int_0^\pi d\sigma\left(\partial_\tau x^{r'}\bar{\gamma}^{r'}\theta_D - \partial_\sigma x^{r'}\bar{\gamma}^{r'}(\theta^1 + \Omega\theta^2) - m x^{r'}\bar{\gamma}^{r'}(\Pi\theta^2 + \Omega\Pi\theta^1)\right); \qquad (4.1)\\ &= -2\sqrt{(2m/\pi)\tanh\tfrac{1}{2}m\pi}\, x_0^{r'}\bar{\gamma}^{r'}\Omega\Pi\hat{\theta}_0 - \sum_{n\neq 0} 2c_n\sqrt{|\omega_n|}a_n^{r'}\bar{\gamma}^{r'}(\Omega + i d_n\Pi)\theta_{-n}. \end{aligned}$$

Let us now include this charge in the D_+ brane algebra containing the bosonic charges $(P^-, P^+, P^r, J^{+r}, J^{rs}, J^{r's'})$, along with the kinematical charges $\hat{Q}^+$ and the worldsheet charges (P_n^I, Q_n). It commutes with the Hamiltonian, transforms as a spinor under J^{rs} and $J^{r's'}$ and anticommutes with $\hat{Q}^+$ to give a central term involving $x_0^{r'}$ as for the D1-brane

[3] In [6] and here we discuss open strings with boundary conditions corresponding to D-branes with no worldvolume flux. It was found in [8] that there are supersymmetric $(+,-,4,0)$ and $(+,-,0,4)$ embeddings that preserve 1/2 of the dynamical supersymmetry but these necessarily involve certain worldvolume fluxes. Actually the $(+,-,4,0)$ and $(+,-,0,4)$ are not even on-shell with respect to the DBI without including this flux. These D-branes with flux were analyzed from the worldsheet point of view in [22, 12].

and the $P_n^{r'}$ and Q_n are supersymmetric partners and P_n^r a singlet with respect to this symmetry.

However, the anticommutator of the charge with itself generates new charges P_0^- and J_0^{IJKL} not contained in the algebra:

$$\{q_0^-, q_0^-\} = \bar{\gamma}^+ P_0^- + J_0^{IJKL}\gamma^{+IJKL}. \tag{4.2}$$

As we have discussed, not only is the Hamiltonian a conserved current, but terms in the Hamiltonian involving the scalars x^I and the fermions are individually conserved on-shell. The specific combination which arises in the anticommutation (4.2) is

$$P_0^{-\tau} = -\tfrac{1}{2}\sum_{r'}\left((\partial_\tau x^{r'})^2 + (\partial_\sigma x^{r'})^2 + m^2(x^{r'})^2 + \frac{1}{8}i(\theta^1\bar{\gamma}^-\partial_\tau\theta^1 + \theta^2\bar{\gamma}^-\partial_\tau\theta^2)\right), \tag{4.3}$$

which evaluated on-shell is

$$P_0^- = -\frac{m}{\pi}\left(\frac{(e^{m\pi}-1)}{(e^{m\pi}+1)}\right)\sum_{r'=p}^{8}(x_0^{r'})^2 - \sum_{r'}\sum_{n>0}\omega_n(a_{-n}^{r'}a_n^{r'} + \tfrac{1}{2}\theta_{-n}\bar{\gamma}^-\theta_n). \tag{4.4}$$

Notice that in the case $p=1$, P_0^- is equal to P^-. In all other cases P_0^- differs from P^- in that all Neumann modes are missing and the coefficient of the fermionic terms is not the same.

The other charges arising are the tensorial symmetries

$$\mathcal{J}_0^{IJKL\tau} = \frac{i}{240}\sum_{r'}\partial_\sigma\theta_N\bar{\gamma}^{r'}\gamma^{-IJKL}\bar{\gamma}^{r'}\theta_D. \tag{4.5}$$

Such tensorial symmetries are also present in the closed string, as was mentioned in [6]. Evaluating the charge in terms of modes we get

$$J_0^{IJKL} = -\frac{1}{480}\sum_{r'}\sum_{n\neq 0}c_n^2\omega_n\theta_{-n}(id_n\Pi + \Omega^t)\bar{\gamma}^{r'}\gamma^{-IJKL}\bar{\gamma}^{r'}(id_n\Pi - \Omega)\theta_n. \tag{4.6}$$

This expression can be further processed using gamma matrix identities to eliminate the summed gamma matrix. The details depend on the brane under consideration and will not be given here. We only note that J_0^{IJKL} vanishes for the D1 brane because $\sum_{r'}\bar{\gamma}^{r'}\gamma^{-r'_1r'_2r'_3r'_4}\bar{\gamma}_{r'} = 0$ (and the D1 brane does not have any worldvolume directions transverse to the lightcone coordinates).

The addition of P_0^- and J_0^{IJKL} to the algebra induces further charges. Explicit calculation gives that

$$[q_0^-, P_0^-] = \frac{1}{8}(1-p)\sum_{r'}\sum_{n\neq 0}2c_n\omega_n\sqrt{|\omega_n|}a_n^{r'}\bar{\gamma}^{r'}(\Omega + id_n\Pi)\theta_{-n}. \tag{4.7}$$

As mentioned above, P_0^- is equal to P^- when $p = 1$. In this case we already know it commutes with q^-, and indeed (4.7) vanishes in this limit. For $p \neq 1$ this commutation hence generates the additional charges

$$q_1^- = \sum_{r'} \sum_{n \neq 0} 2 c_n \omega_n \sqrt{\omega_n} a_n^{r'} \bar{\gamma}^{r'} (\Omega + i d_n \Pi) \theta_{-n}. \tag{4.8}$$

The associated symmetry involves an extra worldsheet derivative, giving rise to the additional factors of ω_n in the mode expansion:

$$q_1^{-\tau} = i \sum_{r'} \left(\partial_\tau x^{r'} \bar{\gamma}^{r'} \partial_\tau \theta_D - \partial_\sigma x^{r'} \bar{\gamma}^{r'} \partial_\tau (\theta^1 + \Omega \theta^2) - m x^{r'} \bar{\gamma}^{r'} \partial_\tau (\Pi \theta^2 + \Omega \Pi \theta^1) \right). \tag{4.9}$$

One can also show that

$$[q_0^-, J_0^{IJKL}] \sim \gamma^{r'} \gamma^{IJKL} \gamma^{r'} q_1^-; \tag{4.10}$$

this is necessary for the Jacobi identity for $[q_0^-, \{q_0^-, q_0^-\}]$ to hold. It is convenient to introduce the notation G_l to denote a charge arising from a symmetry with l additional worldsheet derivatives relative to $G_0 \equiv G$.

To summarize: the addition of q_0^- to the algebra gives the charges P_0^- and J_0^{IJKL} under anticommutation. Commuting these three charges with each other gives q_1^-, and the anticommutation of this charge with itself manifestly gives P_2^- and J_2^{IJKL}. Anticommuting q_0^- with q_1^- generates P_1^- and J_1^{IJKL}. Thus one sees that one cannot add q_0^- to the algebra without adding all of q_l^-, P_l^- and J_l^{IJKL} to close the algebra.

The fermionic generator q_0^- does not depend on any of the Neumann oscillators. There are also fermionic conserved currents that involve only Neumann oscillators. One such current is

$$\tilde{q}_0^{-\tau} = \partial_\tau \partial_\sigma x^r \bar{\gamma}^r \theta_D + \partial_\sigma x^r \bar{\gamma}^r \partial_\sigma \theta_N, \tag{4.11}$$

which gives

$$\tilde{q}_0^- = \sum_{n \neq 0} 2 n c_n \sqrt{\omega_n} a_n^r \bar{\gamma}^r (i\Omega - d_n \Pi) \theta_{-n}. \tag{4.12}$$

This is almost the same mode expansion as q_0^- but differs by factors of n. Anticommuting this operator with itself produces new (higher derivative and tensorial) charges just as in the discussion of q_0^-.

For the other branes where we found additional supersymmetries, a clue to the existence of the additional supercharges was that the spectrum could naturally organize into multiplets of the new supersymmetry. In the case at hand, the massive string states are most naturally organized into multiplets of

$$q = -2\sqrt{(2m/\pi)\tanh\tfrac{1}{2}m\pi}\, x_0^{r'}\bar{\gamma}^{r'}\Omega\Pi\hat{\theta}_0 - \sum_I\sum_{n\neq 0} 2c_n\sqrt{\omega_n}a_n^I\bar{\gamma}^I(i\Omega - d_n\Pi)\theta_{-n}. \tag{4.13}$$

A direct computation yields

$$\{q, q\} = -\bar{\gamma}^+(\Delta H + E_N), \tag{4.14}$$

where E_N and ΔH are given in (3.23). That is, q anticommutes to yield P^- but without the Neumann zero modes. Furthermore, the algebra closes with the addition of only q and $(\Delta H + E_N)$. Note that the absence of Neumann zero modes from this q reflects the fact that states generated by Neumann zero modes do not have the same energy as states generated by any fermionic oscillators. These states should thus not be related by a dynamical supercharge which commutes with the Hamiltonian.

Now the operator q looks close to what one would call a dynamical supercharge, but the issue is whether q and $(\Delta H + E_N)$ are associated with *local* currents. It turns out that both of these currents are non-local. To prove this for the bosonic operator, first note that $(\Delta H + E_N) = (H - E_0)$. H generates the global symmetry $\delta x^+ = \epsilon^+$ but E_0 generates a non-local symmetry. The oscillator expression for E_0 is

$$E_0 = \tfrac{1}{2}\sum_{r=1}^{p-1}((p_0^r)^2 + m^2(x_0^r)^2), \tag{4.15}$$

which generates the field variation

$$\delta x^r = (p_0^r\cos(m\tau) - m x_0^r\sin(m\tau))\epsilon^r. \tag{4.16}$$

To obtain this expression we have used the commutation relations between x^r and the oscillators. The expression is however only meaningful onshell, and does not describe an offshell symmetry.

We can find an offshell symmetry by noticing that this onshell expression can be rewritten as

$$\delta x^r = \frac{1}{\pi}\int_0^\pi d\sigma'\partial_\tau x^r(\tau,\sigma')\epsilon^r. \tag{4.17}$$

This variation is in fact a symmetry of the action offshell: the variation of the action is a total time derivative

$$\delta S = T^2\int d\tau d\sigma d\sigma'(\partial_\tau x^r(\tau,\sigma)\partial_\tau^2 x^r(\tau,\sigma') - m^2 x^r(\tau,\sigma)\partial_\tau x^r(\tau,\sigma'))\epsilon^r; \tag{4.18}$$

$$= T^2\int d\tau d\sigma d\sigma'\partial_\tau(\partial_\tau x^r(\tau,\sigma)\partial_\tau x^r(\tau,\sigma') - m^2 x^r(\tau,\sigma)x^r(\tau,\sigma'))\epsilon^r, \tag{4.19}$$

(recalling that $T = 1/\pi$ for the open string). The associated Noether current is

$$(E_0)^\tau = \tfrac{1}{2}T \int d\sigma' (\partial_\tau x^r(\tau,\sigma)\partial_\tau x^r(\tau,\sigma') + m^2 x^r(\tau,\sigma)x^r(\tau,\sigma')), \tag{4.20}$$

which integrated over sigma gives the conserved charge E_0. E_0 is thence manifestly non-local since it depends on an integration over the worldsheet.

The Dirichlet part of q is exactly q_0^-, and the Neumann part would be equal to $\tilde{q}_0^-$ had the factor of n been absent from the right hand side of (4.12). It appears that there is no *local* current that yields (4.12) without the factor of n. One can, however, proceed as follows. As in the above discussion there are charges $\tilde{q}_l^-$ which are higher derivative variants of $\tilde{q}_0^-$

$$\begin{aligned} \tilde{q}_l^- &= -\frac{1}{\pi}\int_0^\pi d\sigma(\partial_\sigma x^r \bar{\gamma}^r \partial_\tau^{l+1}\theta_D + \partial_\tau\partial_\sigma x^r\bar{\gamma}^r\partial_\tau^{l-1}\partial_\sigma\theta_N), \\ &= \sum_{n\neq 0}(i\omega_n)^l 2nc_n\sqrt{\omega_n}a_n^r\bar{\gamma}^r(i\Omega - d_n\Pi)\theta_{-n}. \end{aligned} \tag{4.21}$$

Let us collect all $\tilde{q}_l^-$ in an infinite-dimensional vector $\vec{\tilde{q}}$ and introduce another (infinite-dimensional) vector $\vec{q}$ whose components are

$$q_n = 2c_n\sqrt{\omega_n}a_n^r\bar{\gamma}^r(i\Omega - d_n\Pi)\theta_{-n}. \tag{4.22}$$

Then (4.21) can be written as

$$\vec{\tilde{q}} = M\vec{q} \tag{4.23}$$

where M is the $(\infty\times\infty)$ matrix

$$M_{ln} = n(i\omega_n)^l. \tag{4.24}$$

One may now solve (4.23) for $\vec{q}$,

$$\vec{q} = M^{-1}\vec{\tilde{q}}. \tag{4.25}$$

(The existence of such a solution depends on M being non-degenerate.) Having obtained the q_n one can then construct q. In this construction, however, we have to use currents with an infinite number of worldsheet derivatives, so q is effectively non-local.

We have given two different arguments for the non-locality of E_0 and q respectively. One can also provide a construction of E_0 similar to the one for q by considering corresponding higher derivative charges and inverting to isolate E_0, and also explicitly demonstrate that the symmetry transformations generated by q are non-local as in the case of the transformations generated by E_0 but we shall not give these details here.

Thus although the D_+ branes preserve fermionic symmetries other than the kinematical supersymmetries, either the corresponding current is non-local or the closure of the algebra on adding one such charge requires the inclusion of currents with an infinite number of worldsheet derivatives. Thus in all cases, these symmetries may be considered non-local. It seems likely that these symmetries will not be respected by the interactions.

We have seen in this section that if we try to include any dynamical supercharges in the algebra for D_+ p-branes with $p \neq 1$ the Jacobi identities induce higher derivative and tensorial currents. There are analogous conserved currents for closed strings and both D_+ and D_- branes, but in other cases we are not forced to consider them; the algebras close without them.

We end this section by giving one more example of such a higher derivative current: the higher derivative variants of the rotational currents for D_+ branes

$$\begin{aligned} (\mathcal{J}^{IJ\tau})^b_{2l} &= (\partial^l_\sigma x^I \partial_\tau \partial^l_\sigma x^J - \partial^l_\sigma x^J \partial^l_\sigma \partial_\tau x^I); \\ (\mathcal{J}^{IJ\tau})^f_{2l} &= -\tfrac{1}{2} i (\partial^l_\sigma \theta^1 \gamma^{-IJ} \partial^l_\sigma \theta^1 + \partial^l_\sigma \theta^2 \gamma^{-IJ} \partial^l_\sigma \theta^2), \end{aligned} \tag{4.26}$$

which evaluated on-shell give

$$\begin{aligned} (J^{IJ})^b_{2l} &= -i \sum_{n>0} n^{2l} (a^I_{-n} a^J_n - a^J_{-n} a^I_n); \\ (J^{IJ})^f_{2l} &= -i m^{2l} \hat{\theta}_0 \gamma^{-IJ} \hat{\theta}_0 - 2i \sum_{n>0} n^{2l} \theta_{-n} \gamma^{-IJ} \theta_n. \end{aligned} \tag{4.27}$$

(Note that when there are Dirichlet zero modes the good symmetries $J^{r's'}_{2l}$ are those given in (4.26) with $x^{r'} \to (x^{r'} - x^{r'}_0)$ just as for the $l = 0$ currents.) Notice that the usual rotational charge is just $J^{IJ} = (J^{IJ})^b_0 + (J^{IJ})^f_0$. One can easily construct analogous higher derivative conserved currents in other cases but we will not present further details here.

5 Brane spectra

Before discussing the spectra in detail it is useful to recall that there are two mass scales in the plane wave. With our conventions, the two scales are the mass m associated with the flux and the string mass, $M_s = 1/\sqrt{\alpha'}$. Setting m to zero yields the flat space limit. Some of the states that were degenerate in flat space now acquire mass splittings of the order of m. In the regime $m \ll M_s$ the string states decouple and one is left with states with mass of order m. These are the states whose dynamics is governed by the DBI which we will call DBI states. The opposite limit, $m \gg M_s$, is the regime where the dual gauge theory operates [3]. Our discussions below follow closely the discussion of the closed string spectrum in [5] along with previous discussions of open string spectra in [9, 23, 24], and so we will emphasize only novel features.

5.1 D_- branes

Let us first discuss the spectra of D_- branes. As is standard we choose the vacuum such that it is annihilated by half of the fermionic and bosonic oscillators. Acting with the other half of the oscillators then creates the space of physical states. The action of the fermion and boson zero modes on the vacuum generates the DBI modes.

For open strings there are eight fermion zero modes which should be divided creation and annihilation operators, such that θ_0^- annihilates the vacuum and θ_0^+ generates the spectrum. However one divides the modes into creation and annihilation operators, one will still generate all the spectrum using θ_0^+ but the energy of the vacuum depends on how one does the split. In [5] it was shown that for the closed string the vacuum energy can vary between 0 and $8m$ according to how divides the zero modes. If one chooses a vacuum with energy greater then zero acting with certain θ_0^+ lowers the energy. For open strings, with half as many fermion zero modes, the energy range has to be $4m$. The preferred split of the fermion modes is that for which the vacuum is the lowest energy state. Noticing that the Hamiltonian contains the term $(-2im\theta_0\bar{\gamma}^-\Omega\Pi\theta_0)$ we find that the most natural choice of vacuum state is

$$\bar{a}_0^r\,|0\rangle = a_n^I\,|0\rangle = \theta_n\,|0\rangle = \theta_0^-\,|0\rangle = 0, \tag{5.1}$$

where we introduce the following projections on the fermion zero modes

$$\theta_0^\pm = \tfrac{1}{2}(1 \pm i\Omega\Pi)\theta_0. \tag{5.2}$$

Then the vacuum is an eigenstate of the lightcone Hamiltonian

$$H_0\,|0\rangle = (\Delta H + \tfrac{1}{2}m(p-5))\,|0\rangle\,. \tag{5.3}$$

There is an overall shift from moving the branes away from the origin, which is given in (3.6), and the other part of the vacuum energy is $-m$, 0 and m for the D3-brane, D5-brane and D7-brane, respectively. The proposal for the lightcone vacuum in the dual defect theory [8] yields precisely these values for the lightcone energy.

The states are also labelled by appropriate quantum numbers that correspond to the conserved angular momenta that commute with the Hamiltonian and among themselves. Furthermore, the eigenstates of the Hamiltonian form supersymmetry multiplets of the q^- supersymmetry. The kinematical supersymmetry generators q^+ do not commute with the

Hamiltonian and are therefore spectrum generating. A detailed analysis of the spectrum and the quantum numbers that each state carries depends on the brane under consideration but the details follow straightforwardly in all cases so we shall only highlight the main features.

The quantum numbers for the vacuum can be computed using the definition (5.1) and the explicit form of J^{IJ}. For the $(+,-,m{+}2,m)$ branes one finds (the result for $(+,-,m,m{+}2)$ branes are evidently similar)

$$\begin{aligned} &(+,-,2,0) \qquad J^{34}\left|0\right\rangle = -\left|0\right\rangle, \quad J^{12}\left|0\right\rangle = J^{r's'}\left|0\right\rangle = 0;\\ &(+,-,3,1) \qquad J^{rs}\left|0\right\rangle = J^{r's'}\left|0\right\rangle = 0;\\ &(+,-,4,2) \qquad J^{56}\left|0\right\rangle = \left|0\right\rangle, \quad J^{rs}\left|0\right\rangle = J^{78}\left|0\right\rangle = 0, \end{aligned} \tag{5.4}$$

which in all cases implies that

$$\{q^-, q^-\}\left|0\right\rangle = 0, \tag{5.5}$$

consistent with the supersymmetry of the vacuum.

The DBI modes are obtained by acting with θ_0^+ and a_0^r on the vacuum, and with our conventions they both raise the energy by m. It is useful to introduce the combinations

$$a_\pm^{rs} = a_0^r \pm i a_0^s. \tag{5.6}$$

Then using (3.9) one finds

$$[H, a_\pm^{rs}] = m a_\pm^{rs}, \qquad [J^{rs}, a_\pm^{rs}] = \pm a_\pm^{rs}. \tag{5.7}$$

Thus $a_\pm^{rs}$ acting on the vacuum raises the energy by m and the J^{rs} charge by ± 1. One may similarly introduce combinations of the fermionic zero modes that transform with definite charge under the rotation charges J^{IJ} by multiplying θ_0 with the projection operators $\mathcal{P}^{IJ} = (1 \pm i\gamma^{IJ})/2$. In the following we shall be schematic. The form of the spectrum is given in Table 1, where we have only listed states with up to five bosonic oscillators but of course there is an infinite number of states. The energy is listed in the left hand column. In the table we use several abbreviated notations. As usual we suppress all spinor indices: by $(\theta_0^+)^2$ we mean $(\theta_0^+)^{[\alpha}(\theta_0^+)^{\beta]}$, etc. By $a^p\left|0\right\rangle$ we mean that we act with p bosonic zero modes $a_\pm^{rs}$; these oscillators can be the same or distinct. We can act on each of the minimum energy states in the left column with an arbitrary number of bosonic zero modes.

Let us consider the first column of table one. The multiplet contains eight bosons and eight fermions. There are six bosons of mass $H_0 + 2m$, one with mass H_0 and another with mass $H_0 + 4m$. The fermions splits into two sets of fours with masses $H_0 + m$ and $H_0 + 3m$, respectively. When $m = 0$ the field content is that of a ten dimensional vector multiplet (or of its appropriate dimensional reduction when viewed from the worldvolume point of view).

$H_0 + 5m$		$(\theta_0^+)^4 a \|0\rangle$	$(\theta_0^+)^3 a^2 \|0\rangle$	$(\theta_0^+)^2 a^3 \|0\rangle$	$(\theta_0^+) a^4 \|0\rangle$	$a^5 \|0\rangle$
$H_0 + 4m$	$(\theta_0^+)^4 \|0\rangle$	$(\theta_0^+)^3 a \|0\rangle$	$(\theta_0^+)^2 a^2 \|0\rangle$	$(\theta_0^+) a^3 \|0\rangle$	$a^4 \|0\rangle$	
$H_0 + 3m$	$(\theta_0^+)^3 \|0\rangle$	$(\theta_0^+)^2 a \|0\rangle$	$(\theta_0^+) a^2 \|0\rangle$	$a^3 \|0\rangle$		
$H_0 + 2m$	$(\theta_0^+)^2 \|0\rangle$	$(\theta_0^+) a \|0\rangle$	$a^2 \|0\rangle$			
$H_0 + m$	$(\theta_0^+) \|0\rangle$	$a \|0\rangle$				
H_0	$\|0\rangle$					

Table 1: DBI spectrum for D_- branes

Recall that in flat space one considers a vacuum with momentum $|k\rangle$ and builds on it the massless states that carry this momentum, or equivalently localized at same spacetime point x in the x-space representation. In the plane wave background the momentum is not a good quantum number as it does not commute with the Hamiltonian. Instead the bosonic zero modes reflect the existence of a quadratic (harmonic oscillator) potential in the Neumann directions. Acting with the bosonic creation modes allows the string to "climb up" the potential: it is localized further from the origin in the Neumann directions.

It is worth mentioning that in the case of $(+,-,4,2)$ branes the states $(a_+^{56})^p|0\rangle$ are supersymmetric states which carry p units of J^{56} charge. It follows from (3.15) that

$$\{q^-, q^-\}(a_+^{56})^p|0\rangle = 0. \tag{5.8}$$

It would be interesting to explore systematically supersymmetric states and their dual interpretations.

Let us note that the only difference between the spectra of branes with different Dirichlet boundary conditions is that states are labelled by the appropriate $\hat{H}$ and their eight dynamical supercharges $\hat{q}^-$ do not in general descend from the closed string.

Since the kinematic supercharges are essentially proportional to the fermion zero modes ($q^+ \sim \bar{\gamma}^-\theta_0$), the action of the q^+ takes us up and down the columns in the diagram. This is of course consistent with the fact that acting with q^+ necessarily raises or lowers the energy by m. Note that we go down the columns and lower the energy by acting with the fermion annihilation operators θ_0^-.

Let us consider the action of the dynamical supercharges. With our choice of vacuum, non-zero modes in q^- annihilate every state in the DBI multiplet. Thus the only non-trivial action by the supercharge arises from the zero mode part

$$q_0^- = 2\sqrt{2m}(\bar{a}^r\bar{\gamma}^r\theta_0^+ + a^r\bar{\gamma}^r\theta_0^-). \tag{5.9}$$

This formula should be contrasted with the flat space case $q^- = 2k^r \bar{\gamma}^r \theta_0$, where k is the momentum. The vacuum is a singlet under the q^- supersymmetry. For the other states q^- acts by moving us along the rows in the diagram. This is consistent with the fact that q^- commutes with the Hamiltonian.

Now let us consider the non-zero mode part of the spectrum. This is obtained by acting with (θ_{-n}, a^I_n), or equivalently the conserved worldsheet charges (Q_{-n}, P^I_n), on the vacuum, and then acting on the resulting states with zero modes. Part of the spectrum is given in Table 2.

$H_0 + \omega_n + m$	$a a^I_{-n} \|0\rangle$	$(\theta_0^+) a^I_{-n} \|0\rangle$		$(\theta_0^+)\theta_{-n} \|0\rangle$	$\theta_{-n} a \|0\rangle$
$H_0 + \omega_n$		$a^I_{-n} \|0\rangle$		$\theta_{-n} \|0\rangle$	
H_0			$\|0\rangle$		

Table 2: Stringy modes for D_- branes

The kinematical supersymmetry acts vertically in this diagram, whilst the dynamical supersymmetry acts horizontally: it relates states with the same number of mode n oscillators. The spectrum will thus form representations of both the dynamical and the kinematical supersymmetry. Since there are eight bosonic and eight fermionic oscillators at each n, the total number of states with no bosonic zero modes at this level is 256. Note that this is true for all n, whereas in flat space single oscillator states $a^I_{-n} |0\rangle$ has the same energy as multiple oscillator states $\prod_i a^I_{-n_i} |0\rangle$, $\sum_i n_i = n$, and so the number of states at each level grows exponentially with n. This is not true in the plane wave since the frequencies ω_{n_i} are never rationally related.

Let us emphasize again that the key differences compared to the spectrum in flat space are that states are labelled by the number of boson zero modes rather than the momentum, and the DBI states have energies of order m. In the regime $m \ll M_s$ the latter decouple from string modes and the dynamics is described by the DBI multiplet. We will see in the next section that the spectrum of fluctuations around the D-brane embedding reproduces what we have just found. In the opposite limit, $m \gg M_s$,

$$\omega_n = \sqrt{n^2 + m^2} = m(1 + \frac{n^2}{2m^2} + \cdots) \tag{5.10}$$

and low-lying string states have the same energy as massive DBI modes. This is the limit where the dual gauge theory operates [3].

5.2 D_+ branes

Let us now consider the spectrum for D_+ branes. In this case the Hamiltonian is independent of the fermionic zero modes, and the vacuum state is degenerate (as in flat space). We choose the vacuum so that

$$\bar{a}_0^r \left|0\right\rangle = a_n^I \left|0\right\rangle = \theta_n \left|0\right\rangle = \mathcal{P}_- \hat{\theta}_0 \left|0\right\rangle = 0, \tag{5.11}$$

where we use projections $\mathcal{P}_\pm \hat{\theta}_0 = \pm \hat{\theta}_0$ to divide the zero mode fermions into two sets of four. (The form of the projectors is to a large extend arbitrary and it does not influence the discussion below). The vacuum energy is then

$$H_0 = \tfrac{1}{2} m(p-1) + \Delta H. \tag{5.12}$$

ΔH again represents the shift in energy from moving the branes away from the origin. For branes located at the origin, the energy is 0, m, $2m$, $3m$ and $4m$ for D1-branes, D3-branes, D5-branes, D7-branes and D9-branes respectively. We now use $\mathcal{P}_+\hat{\theta}_0$ and a_0^r to create the DBI spectrum.

Let us first discuss the D1 brane. In this case there are no bosonic zero modes and so we have only sixteen states rather than an infinite number of states. The lightcone energy

$\left\|0\right\rangle$	$(\mathcal{P}_+\hat{\theta}_0)\left\|0\right\rangle$	$(\mathcal{P}_+\hat{\theta}_0)^2\left\|0\right\rangle$	$(\mathcal{P}_+\hat{\theta}_0)^3\left\|0\right\rangle$	$(\mathcal{P}_+\hat{\theta}_0)^4\left\|0\right\rangle$

Table 3: DBI spectrum for $D1$ branes

of these states is zero, unless the D1 brane is located away from the origin. It will be convenient to denote as $|0;\theta_0^+\rangle$ the supermultiplet in Table 3. The other D_+ branes have bosonic zero modes, so the DBI spectrum is infinite: one acts on $|0;\theta_0^+\rangle$ with the bosonic zero modes. Each bosonic oscillator increases the lightcone energy, so at energy $H_0 + m$ we have the states $a|0;\theta_0^+\rangle$, at energy $H_0 + 2m$ the states $a^2|0;\theta_0^+\rangle$, etc.

Thus the spectrum is rather different to that of the D_- branes. The lowest energy level now consists of eight bosons and eight fermions, as in flat space. The action of the supercharges is also rather different. Firstly in this case we do not have kinematical supercharges descending from the closed string. We do however have the conserved charges $\hat{Q}^+$. These commute with the Hamiltonian and move us between states in the same row of the diagram.

The only case in which we have dynamical supercharges descending from the closed string is the D1-brane. This brane is already special in that there are no bosonic zero modes, and so we have only sixteen states rather than an infinite number. q^- acting on these states vanishes unless the brane is displaced from the origin, and acts between states in the single row in the diagram. For example,

$$q^- \left|0\right\rangle = -2\sqrt{(2m/\pi)\tanh\tfrac{1}{2}m\pi}\, x_0^{r'}\bar{\gamma}^{r'}\Pi'\mathcal{P}_+\hat{\theta}_0 \left|0\right\rangle . \tag{5.13}$$

The rest of the spectrum is obtained by acting with the non zero modes on the vacuum; this is illustrated for the D1-brane in Table 4.

$H_0 + \omega_n$	$a^I_{-n}\lvert 0;\theta_0^+\rangle$	$\theta_{-n}\lvert 0;\theta_0^+\rangle$
H_0	$\lvert 0;\theta_0^+\rangle$	

Table 4: Stringy spectrum for D1-brane. $|0;\theta_0^+\rangle$ denotes the zero mode supermultiplet.

For the D1-brane the states form representations of the surviving dynamical supersymmetry; again this relates states with the same number of mode n oscillators. Since there are eight bosonic oscillators at each level, and θ_{-n} has eight independent components, the total number of states at the first excited level in the diagram is $(16 \times 8 + 16 \times 8) = 256$. This is the same as for the D_- branes and the same as in flat space. For the other D_+ branes, however, although there are still eight bosonic and eight fermionic oscillators at each level, these do not, following the analysis of the previous section, form representations of a local supercharge except in the flat space limit. Most likely, loop corrections will lift the degeneracy between bosons and fermions.

6 DBI fluctuation spectrum

The open string states obtained by acting with fermionic and bosonic zero mode operators on the open string vacuum should be in one to one correspondence with the fluctuation modes of DBI fields expanded about the embeddings found in [8]. The aim of this section is to find the form of the bosonic DBI equations of motion expanded to linear order in fluctuations and then to determine the corresponding light cone energy spectrum. We treat only the bosonic modes which can be dealt with very simply; the analysis of the fermionic terms is more complicated and will not be attempted here.

For definiteness we will consider in detail the following two cases. The first is D_- 3-branes, both static and rotating, whilst the second is D1-branes. The case of the D_- 7-brane at the origin was discussed in [9]. Whilst it is straightforward to derive the spectrum for the other D-branes, we do have to treat each case separately because the fluctuation field equations do differ between cases, particularly in the couplings to the RR flux.

Let us first give the worldvolume action for a Dp-brane:

$$\begin{aligned} I_p &= I_{DBI} + I_{WZ}; \\ I_{DBI} &= = -T_p \int_M d^{p+1}\xi e^{-\Phi} \sqrt{-\det(g_{ij} + \mathcal{F}_{ij})}; \qquad I_{WZ} = T_p \int_M e^{\mathcal{F}} \wedge C, \end{aligned} \tag{6.1}$$

with T_p the Dp-brane tension. Here ξ^i are the coordinates of the $(p+1)$-dimensional worldvolume M which is mapped by worldvolume fields X^μ into the target space which has (string frame) metric $g_{\mu\nu}$. This embedding induces a worldvolume metric $g_{ij} = g_{\mu\nu}\partial_i X^\mu \partial_j X^\nu$. The worldvolume also carries an intrinsic abelian gauge field A with field strength F. $\mathcal{F} = F - B$ is the gauge invariant two-form with $B_{ij} = \partial_i X^\mu \partial_j X^\nu B_{\mu\nu}$ the pullback of the target space NS-NS 2-form. Note that we have set $2\pi\alpha' = 1$. The RR n-form gauge potentials (pulled back to the worldvolume) are collected in $C = \bigoplus_n C_{(n)}$ and the integration over the worldvolume automatically selects the proper forms in this sum. In what follows it is convenient to use the covariant equations of motion following from this action which were given in [8].

6.1 D_- 3-brane

For a D3-brane in the plane wave background the appropriate equations are [8]

$$\partial_i(\sqrt{-M}\theta^{ii_1}) = 0; \tag{6.2}$$

$$\frac{1}{4!}\epsilon^{i_1..i_4} F_{i_1 i_2 i_3 i_4 \mu} = -\partial_i(\sqrt{-M}G^{ij}\partial_j X^\nu g_{\mu\nu}) + \tfrac{1}{2}\sqrt{-M}(G^{ij}\partial_i X^\nu \partial_j X^\rho g_{\nu\rho,\mu}), \tag{6.3}$$

where G^{ij} and θ^{ij} are the symmetric and antisymmetric parts of M^{ij}, respectively (M^{ij} is the inverse of $M_{ij} = g_{ij} + \mathcal{F}_{ij}$). The first equation is the gauge field equation, whilst the second encapsulates the scalar field equations. Recall that in our conventions the five form flux in the background is $F_{+1234} = F_{+5678} = 4\mu$.

6.1.1 Static D_- 3-brane

Let us linearize these equations about the static $(+,-,2,0)$ embedding, for which the transverse coordinates are fixed constants: we set $\xi^i = (x^+, x^-, x^1, x^2)$ and $X^{r'} = x_0^{r'} + x^{r'}$. One finds that the fluctuations in these transverse scalars satisfy the following equations:

$$\Box x^3 = 4\mu\partial_- x^4; \quad \Box x^4 = -4\mu\partial_- x^3; \quad \Box x^{i'} = 0 \tag{6.4}$$

where

$$\Box = \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}g^{ij}\partial_j) = (2\partial_+\partial_- + \mu^2((x^1)^2 + (x^2)^2 + \sum_{r'}(x_0^{r'})^2)\partial_-^2 + \partial_1^2 + \partial_2^2). \tag{6.5}$$

(g_{ij} is the background induced metric). Here r' runs over all transverse scalars and i' runs from 5 to 8. The additional terms in the (x^3, x^4) equations arise from the couplings to the RR background. Note that to linearized order the scalar and gauge field fluctuations are not coupled and only the (x^3, x^4) fluctuations mix with each other. We will give the gauge field equations below. To find the lightcone spectrum it is useful to Fourier transform in the (x^-, x^1, x^2) directions:

$$\psi(x^+, x^-, x^1, x^2) = \int \frac{dp^+ dp^1 dp^2}{(2\pi)^{\frac{3}{2}}} e^{i(p^+x^- + p^1x^1 + p^2x^2)} \tilde{\psi}(x^+, p^+, p^1, p^2). \tag{6.6}$$

Then the scalar field equations take the form

$$\Box_p \tilde{x}^3 = 4im\tilde{x}^4; \quad \Box_p \tilde{x}^4 = -4im\tilde{x}^3; \quad \Box_p \tilde{x}^{i'} = 0 \tag{6.7}$$

where

$$\Box_p = (2H + m^2(\partial^2_{p^1} + \partial^2_{p^2} - \sum_{r'}(x_0^{r'})^2)) - (p^1)^2 - (p^2)^2), \tag{6.8}$$

Here $H = ip^+\partial_+$ may be interpreted as the lightcone Hamiltonian (the factor of p^+ is due to the normalization of generators introduced in (B.1)). Introducing a complex scalar ϕ such that $\tilde{\phi} = \tilde{x}^3 + i\tilde{x}^4$ and

$$\Box_p \tilde{\phi} = 4m\tilde{\phi}; \quad \Box_p \bar{\tilde{\phi}} = -4m\bar{\tilde{\phi}}, \tag{6.9}$$

we find that

$$\begin{aligned} H_{\tilde{\phi}} &= 3m + h; \quad H_{\bar{\tilde{\phi}}} = -m + h; \quad H_{\tilde{x}^{i'}} = m + h, \\ h &= m(a^u\bar{a}^u + \tfrac{1}{2}m\sum_{r'}(x_0^{r'})^2). \end{aligned} \tag{6.10}$$

To get to these expressions we write each H in terms of the momenta and then introduce the standard creation and annihilation operators

$$a^u = \frac{1}{\sqrt{2m}}(p^u - m\partial_{p^u}), \quad \bar{a}^u = \frac{1}{\sqrt{2m}}(p^u + m\partial_{p^u}), \quad [\bar{a}^u, a^v] = \delta^{uv}, \tag{6.11}$$

where $u, v = 1, 2$. After normal ordering, the Hamiltonians for each of the six scalars are as given above. As usual the spectrum of states will be obtained by acting with a^u on vacua satisfying $\bar{a}^u \left|0\right\rangle = 0$. The lowest lightcone energy value for each mode is given by

$$\begin{aligned} E_{\phi} &= 3m + \Delta H; \quad E_{\bar{\phi}} = -m + \Delta H; \quad E_{x^{i'}} = m + \Delta H; \\ \Delta H &= \tfrac{1}{2}m^2\sum_{r'}(x_0^{r'})^2. \end{aligned} \tag{6.12}$$

The analysis for the gauge field is rather similar. In the lightcone $A_- = 0$, and Lorentz gauge, $\partial_- A_+ + \partial_1 A_1 + \partial_2 A_2 = 0$, A_+ is completely determined in terms of the modes A_1 and A_2,

$$\Box A_+ = 2\mu^2 \partial_-(x^u A_u) \tag{6.13}$$

and the Fourier transforms of A_1 and A_2 satisfy the equations

$$\Box_p \tilde{A}_u = 0. \tag{6.14}$$

Thus using the analysis above the lightcone Hamiltonian for the gauge field modes is

$$H_{\tilde{A}} = m(a^u \bar{a}^u + 1 + \tfrac{1}{2} m(x_0^{r'})^2), \tag{6.15}$$

and the lowest lightcone energy is thus $E_A = m + \Delta H$.

Now let us compare this to the bosonic part of the zero mode spectrum we found from the open strings. Consider first the case where the transverse positions are zero. Then the DBI analysis tells us that the lowest energy states are one complex scalar with energy $-m$, four scalars and the physical components of the gauge field with energy m and the complex conjugate scalar with energy $3m$. This is indeed in agreement with what we found from the open strings.

One may also compute the rotation charges of the fluctuations. For example, J^{34} is realized as a differential operator as

$$J^{34} = -i(x^3 \partial_4 - x^4 \partial_3). \tag{6.16}$$

It thus follows that the $\bar{\phi}$, the fluctuation with the lowest lightcone energy, has J^{34} charge -1, in agreement with the open string computation.

It is also worth noting that the defect operator dual to the light-cone vacuum [8] also carries the same rotational charges. This can be read off from Table 1 of [25]. The J_{23} assignments in that paper correspond to the J^{34} assignments here, and the R-symmetry $SO(4)$ corresponds to $SO(4)'$. The operator under consideration is given in (9.1) of [8] or (6.1) of [25]. Using Table 1 one finds that $J^{34} = -1$ for this operator. Furthermore, the operator is a singlet under the transverse $SO(4)$ which is also in agreement with the fact that the light-cone vacuum is annihilated by $J^{r's'}$. The dimension of the defect operator saturates the BPS bound of the superconformal algebra, and the lightcone vacuum saturates a corresponding bound that can be read off from the superalgebra in (3.16). Finally, both of them appear to be subtle in the following sense: the lightcone vacuum appears to be tachyonic, i.e. it has a negative lightcone energy, and, as was recently pointed out in

[25], the dual operator may suffer from strong infrared effects since its constituents contain (undifferentiated) $2d$ massless scalars. Since the D3 brane is supersymmetric, we expect that it is stable. We leave further understanding of these issues for future work.

When any of transverse positions are shifted from zero, there is a shift in the energy of each of these states by $\frac{1}{2}m^2(x_0^{r'})^2$. At first sight this seems to disagree with the open string result, where we found that the energy was instead shifted by

$$\Delta H = \frac{m}{\pi}\frac{(e^{m\pi}-1)}{(e^{m\pi}+1)}(x_0^{r'})^2. \tag{6.17}$$

This discrepancy is however resolved by recalling that the DBI captures only the zero slope limit of the open strings and in this limit $m \to 0$. In this limit (6.17) indeed gives $\Delta H = \frac{1}{2}m^2(x_0^{r'})^2$, in agreement with the DBI result.

6.1.2 Symmetry related D_- 3-branes

Let us now consider the spectrum for symmetry-related D3-branes, for which the transverse scalars about which we linearize satisfy (2.11). The Laplacian for branes with time dependent Dirichlet boundary conditions is

$$\Box = \left(2\partial_+\partial_- + \mu^2((x^1)^2 + (x^2)^2 + \sum_{r'}((x_0^{r'})^2 - \mu^{-2}(\partial_+ x_0^{r'})^2))\partial_-^2 + \partial_1^2 + \partial_2^2\right), \tag{6.18}$$

and for the boundary conditions (2.11) we have

$$\sum_{r'}((x_0^{r'})^2 - \mu^{-2}(\partial_+ x_0^{r'})^2) = (a^2 - b^2)\cos(2\mu x^+), \tag{6.19}$$

where $a^2 = \sum_{r'}(a^{r'})^2$. An analysis similar to the one described for the static branes yields the same equations (6.4) but with the Laplacian in (6.18). The most complicated part is to check that the equation (6.3) with $\mu = +$ is satisfied.

Recall that the differential form for the symmetries is:

$$\begin{aligned} P^+ &= -i\partial_-; \quad P^- = -ip^+\partial_+; \\ P^I &= -i\sqrt{p^+}\left(\cos(\mu x^+)\partial_I + \mu\sin(\mu x^+)x^I\partial_-\right); \\ J^{+I} &= -i(\sqrt{p^+})^{-1}\left(\mu^{-1}\sin(\mu x^+)\partial_I - \cos(\mu x^+)x^I\partial_-\right), \end{aligned} \tag{6.20}$$

where the factors of i are because these are operators and we include appropriate factors of p^+ to correspond with the normalizations given in (B.1).

Thus the time dependent terms in (6.19) can be rewritten in terms of $P^{r'}$ and $J^{+r'}$ to give

$$\Box = \left(-2(P^- + \mu\sqrt{p^+}\sum_{r'}(b^{r'}P^{r'} - ma^{r'}J^{+r'}) - \tfrac{1}{2}m(a^2+b^2)) + ...\right), \tag{6.21}$$

where the ellipses denote the terms involving (p^1, p^2) in (6.8). Thus if we define an operator

$$\tilde{P}^- = P^- + \mu\sqrt{p^+}\sum_{r'}(b^{r'}P^{r'} - ma^{r'}J^{+r'}) - \tfrac{1}{2}m(a^2 + b^2), \tag{6.22}$$

then this operator will have precisely the same eigenvalues as for the static brane P^-. This reproduces the results of section (3.4). The gauge field equation can be analyzed similarly.

6.2 D_+ 1-brane

An analogous analysis holds for the bosonic spectrum of the D1-brane. The equations of motion for the eight transverse scalars are

$$0 = -\partial_i(\sqrt{-M}G^{ij}\partial_j X^\nu g_{\mu\nu}) + \tfrac{1}{2}\sqrt{-M}(G^{ij}\partial_i X^\nu \partial_j X^\rho g_{\nu\rho,\mu}). \tag{6.23}$$

Linearizing about $\xi^i = (x^+, x^-)$ with $X^I = x_0^I + x^I$, where x_0^I is constant, the equations for the eight transverse scalars all satisfy

$$\Box x^I = 0, \tag{6.24}$$

where now

$$\Box = (2\partial_+\partial_- + \mu^2\sum_{I=1}^{8}(x_0^I)^2\partial_-^2). \tag{6.25}$$

After Fourier transforming we can identify the lightcone Hamiltonian for each mode as

$$H_{x^I} = \tfrac{1}{2}m^2\sum_{I=1}^{8}(x_0^I)^2. \tag{6.26}$$

In this case there are no creation and annihilation operators and the entire massless bosonic spectrum consists of just these eight states, whose energy is zero when the transverse positions are zero and is shifted when the transverse positions are non zero. By the same arguments as above, in the $m \to 0$ limit this shift agrees with the open string result. The analysis of symmetry related D1-branes follows as in the previous subsection, and we will not give the details here.

7 Boundary States

The aim of this section is to discuss boundary states corresponding to the closed string descriptions of the branes discussed here. Boundary states for the plane wave background have been discussed in [7, 10, 12]. The analysis in these papers followed closely the flat space analysis in [18]. In particular, in these works the fermionic gluing conditions were determined by requiring that the boundary state was annihilated by combinations of the target space supersymmetry. We shall follow instead the analysis of boundary states in the RNS formalism initiated in [19, 20, 21] and obtain the gluing conditions from the boundary conditions of the worldsheet fields. The symmetries that are preserved by the boundary state are determined afterwards.

As is well-known one loop open string diagrams that connect two D-branes can also be viewed as tree level propagation of a closed string emitted from one brane and absorbed by the second. In the two descriptions the roles of the τ and σ coordinates are exchanged. This means in particular that if we choose the light-cone gauge in the closed string description, then the lightcone time is a direction transverse to the brane and the branes are (m, n) instantonic branes. These branes are related via open-closed duality to $(+, -, m, n)$ branes. We refer to [10, 12] for a detailed discussion of these points. These papers also address the issue of consistency of the two descriptions. In particular, [10] checked that the Cardy condition holds for static D_- branes located at the origin whilst [12] checked this condition for D_+1 branes.

The emphasis in this section is on the derivation of the gluing conditions for the boundary states and their symmetries. We work throughout with Lorentzian signature, but one should consider appropriate Wick rotations for the fields to satisfy appropriate reality conditions.

7.1 Gluing conditions

To construct the boundary state we will follow closely the discussion of boundary states in the RNS formalism [19, 20, 21]. Recall that the boundary state is a closed string state that represents the addition of a boundary to the tree level worldsheet. The boundary state is constructed by imposing the boundary conditions of the worldsheet fields as operator relations. The appropriate boundary is now spacelike: at time $\tau = \tau_0$ a closed string is created (or annihilated) from the vacuum.

In [6] we have worked out the variations of the worldsheet action, see (2.6)-(2.10). For boundaries at fixed $\tau = \tau_0$, we find that appropriate boundary conditions corresponding to static branes in lightcone gauge are,

$$\begin{gathered}\partial_\sigma x^-| = 0; \quad \partial_\sigma x^+| = 0; \quad \partial_\tau x^r| = 0; \quad \partial_\sigma x^{r'}| = 0; \\ (\delta\theta^1\bar{\gamma}^-\theta^1 + \delta\theta^2\bar{\gamma}^-\theta^2)| = 0.\end{gathered} \tag{7.1}$$

Recall that σ is now tangential to the boundary and so $\partial_\sigma x^m| = 0$ is a Dirichlet condition whilst $\partial_\tau x^m| = 0$ is a Neumann condition. The conditions on the bosons allow the choice of lightcone gauge $x^+ = p^+\tau$. The fermion condition can be satisfied by choosing

$$(\theta^1 + iM\theta^2)| = 0, \tag{7.2}$$

where M is an orthogonal matrix, the relevant choice of which is the product of gamma matrices over the Neumann directions.

As usual, the x^- boundary condition is automatically implemented by the Virasoro constraint. The relevant constraint in lightcone gauge is

$$p^+\partial_\sigma x^- + i(\theta^1\bar{\gamma}^-\partial_\sigma\theta^1 + \theta^2\bar{\gamma}^-\partial_\sigma\theta^2) + \partial_\tau x^I \partial_\sigma x^I = 0, \tag{7.3}$$

which with the conditions in (7.1) enforces that x^- is Dirichlet.

We now define the boundary state by implementing these conditions as operator identities on the boundary state, using the closed string mode expansions. For a Neumann scalar x^r we enforce

$$\partial_\tau x^r \left|B\right\rangle_{\tau=\tau_0} = 0 \tag{7.4}$$

which in terms of modes becomes

$$\left(p_0^r \cos(m\tau_0) - m x_0^r \sin(m\tau_0) - \sum_{n\neq 0}(\alpha_n^{1I} e^{-i(\omega_n\tau_0 + n\sigma)} + \alpha_n^{2I} e^{-i(\omega_n\tau_0 - n\sigma)})\right) \left|B\right\rangle_{\tau=\tau_0} = 0. \tag{7.5}$$

Since this holds for all σ, it imposes the conditions

$$\begin{aligned} (p_0^r - m\tan(m\tau_0)x_0^r)\left|B\right\rangle_{\tau=\tau_0} &= 0; \\ (\alpha_n^{1I} e^{-i\omega_n\tau_0} + \alpha_{-n}^{2I} e^{i\omega_n\tau_0})\left|B\right\rangle_{\tau=\tau_0} &= 0. \end{aligned} \tag{7.6}$$

These conditions depend explicitly on τ_0. This is because the boundary state breaks translational invariance along the time direction, and the defining conditions do not in general commute with the Hamiltonian. The first condition in (7.6) differs from that used in [10] which was

$$p_0^r \left|B\right\rangle_{\tau=\tau_0} = 0. \tag{7.7}$$

As pointed out in [10] this boundary condition is not consistent with x^- being pure Dirichlet (using the Virasoro constraint) except at $\tau = 0$. Imposing (7.6) instead, x^- is a Dirichlet coordinate.[4] Having made the point that the boundary conditions depend explicitly on τ_0, let us restrict for simplicity to $\tau_0 = 0$.

[4]We should note however that (7.7) is still consistent with the variational problem in lightcone gauge. When we modify the conditions in (7.1) so that the bosonic conditions are neither pure Neumann nor pure Dirichlet the Virasoro constraint implies that $p^+\partial_\sigma x^-| = -\partial_\tau x^I \partial_\sigma x^I|$. This condition can be restated as a coupling of the boundary variations $p^+\delta x^-| = -\partial_\tau x^I \delta x^I|$, which is sufficient to remove boundary terms in the variational problem following the analysis of [6].

The corresponding Dirichlet conditions for $x_0^{r'} = q^{r'}$ on the boundary are

$$q^{r'}\,|B\rangle_0 = \left(x_0^{r'} + i\sum_{n\neq 0} \omega_n^{-1}(\alpha_n^{1r'} e^{-in\sigma} + \alpha_n^{2r'} e^{in\sigma}) \right) |B\rangle_0\,, \tag{7.8}$$

which is satisfied by imposing

$$\begin{aligned} x_0^{r'}\,|B\rangle_0 &= q^{r'}\,|B\rangle_0\,; \\ (\alpha_n^{1r'} - \alpha_{-n}^{2r'})\,|B\rangle_0 &= 0. \end{aligned} \tag{7.9}$$

Now let us discuss the fermionic conditions; as in the open string analysis it is convenient to divide the discussion into D_- branes for which $(M\Pi)^2 = -1$ and D_+ branes for which $(M\Pi)^2 = 1$. Then the defining conditions for D_- branes are

$$\begin{aligned} (\theta_0^1 + iM\theta_0^2)\,|B\rangle_0 &= 0; \\ (\theta_{-n}^1 + iM\theta_n^2)\,|B\rangle_0 &= 0. \end{aligned} \tag{7.10}$$

These are exactly as in flat space, and are also the conditions discussed in [7, 10].

However, the defining conditions for D_+ branes coming from the mode expansion are

$$\begin{aligned} (\theta_0^1 + iM\theta_0^2)\,|B\rangle_0 &= 0; \\ (\theta_{-n}^1 + in^{-1}(M\omega_n + m\Pi)\theta_n^2)\,|B\rangle_0 &= 0, \end{aligned} \tag{7.11}$$

the latter of which differs from that in flat space. This condition has been recently discussed in [12]. Note the close relationship between this expression, and the relation between θ_n and $\tilde{\theta}_n$ appearing in the open string D_+ brane mode expansions. One should note that the factors of i in these expressions are necessary for consistency with the commutation relations: the operators appearing here manifestly anticommute with themselves. Furthermore, the zero mode condition commutes with the Hamiltonian only for the D_+ branes.

It is straightforward to construct the boundary state given the gluing conditions presented in this section. Explicit expressions (for some of them) can be found in [7, 10, 12]. We will not need these expressions, however, so we will not present them here.

7.2 Symmetries

Having defined the boundary state one may now check what symmetries it preserves. This can be done using the explicit expressions for the closed string generators in terms of modes. The relevant formulas, derived in [5], are reviewed in appendix C.

The fermionic conditions (7.10) and (7.11) imply that the static branes preserve the same number of supersymmetries as found by our open string analysis. To see this, note first that the zero mode conditions in (7.10) and (7.11) can immediately be rewritten as

$$(Q^{+2} - iMQ^{+1})\,|B\rangle_0 = 0, \tag{7.12}$$

and thus in both cases the boundary state is annihilated by eight kinematical supercharges. Let us emphasize again that only in the (7.11) case does this condition commute with the Hamiltonian.

To find the number of dynamical charges which annihilate each boundary state we need to use the mode expansions of Q^- along with the bosonic and fermionic gluing conditions. For the D_- branes this analysis was carried out in [7] and it was found that the condition

$$(Q^{-1} + iMQ^{-2})\,|B\rangle_0 = 0 \tag{7.13}$$

was satisfied provided that the Dirichlet transverse positions $q^{r'} = 0$.

Now consider D_--branes displaced from the origin by Dirichlet transverse positions $q^{r'}$. As in our open string analysis, the obstruction to the condition (7.13) being satisfied is the Dirichlet zero modes. However, one may verify that the boundary state satisfies

$$\left(Q^{-1} + iMQ^{-2} - \tfrac{1}{2} i\mu\sqrt{p^+}\sum_{r'} q^{r'}\gamma^{+r'} M\Pi(Q^{+2} + iMQ^{+1})\right)|B\rangle_0 = 0, \tag{7.14}$$

in addition to (7.12) and is thus annihilated by sixteen supercharges. This condition looks rather different to the open string analysis but can be understood as follows. The translational symmetries act as

$$\delta x^- = \sqrt{p^+}\mu\sin(\mu x^+)\epsilon^I x^I; \qquad \delta x^I = \sqrt{p^+}\cos(\mu x^+)\epsilon^I, \tag{7.15}$$

and hence on the hypersurface $x^+ = 0$ simply act as constant shifts of the coordinates x^I. Thus in the closed string sector branes located at $\tau = 0$, x^- constant, $x^{r'} = 0$ are related by the broken translational symmetries to those at constant Dirichlet positions, $x^{r'} = q^{r'}$.

Finally, let us consider the D-instanton. Using in particular the second condition in (7.11) along with bosonic conditions one finds that

$$(Q^{-1} + iMQ^{-2})\,|B\rangle_0 = 0 \tag{7.16}$$

is satisfied for the D-instanton, and it thus also preserves 16 supersymmetries. One may verify that (7.16) is not satisfied by the other D_+ branes. As in the open string analysis, the obstruction is the Neumann modes.

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A Conventions

We follow closely the conventions of [4]. The Dirac matrices in ten dimensions Γ^μ are decomposed in terms of 16-dimensional gamma matrices γ^μ such that

$$\Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \bar{\gamma}^\mu & 0 \end{pmatrix}, \tag{A.1}$$

where

$$\gamma^\mu \bar{\gamma}^\nu + \gamma^\nu \bar{\gamma}^\mu = 2\eta^{\mu\nu}, \quad \gamma^\mu = (\gamma^\mu)^{\alpha\beta}, \quad \bar{\gamma}^\mu = \gamma^\mu_{\alpha\beta}, \tag{A.2}$$

$$\gamma^\mu = (1, \gamma^I, \gamma^9), \quad \bar{\gamma}^\mu = (-1, \gamma^I, \gamma^9). \tag{A.3}$$

Here (α, β) are $SO(9,1)$ spinor indices in chiral representation; we use the Majorana representation for Γ such that $C = \Gamma^0$, and so all γ^μ are real and symmetric. We use the convention that $\gamma^{\mu_1 \cdot\cdot \mu_k}$ are the antisymmetrised product of k gamma matrices with unit strength. $\gamma^\mu, \gamma^{\mu_1\mu_2\mu_3\mu_4}$ and $\gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5}$ are symmetric and $\gamma^{\mu_1\mu_2}$ and $\gamma^{\mu_1\mu_2\mu_3}$ are antisymmetric matrices.

We assume the normalization $\gamma^0\bar{\gamma}^1 ... \gamma^8\bar{\gamma}^9 = 1$, so that

$$\Gamma_{11} = \Gamma^0 ... \Gamma^9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.4}$$

We define

$$\Pi^\alpha{}_\beta \equiv (\gamma^1\bar{\gamma}^2\gamma^3\bar{\gamma}^4)^\alpha{}_\beta, \quad (\Pi')^\alpha{}_\beta \equiv (\gamma^5\bar{\gamma}^6\gamma^7\bar{\gamma}^8)^\alpha{}_\beta, \quad \gamma^\pm = \bar{\gamma}^\mp = \frac{1}{\sqrt{2}}(\gamma^9 \pm 1), \quad \gamma^0\bar{\gamma}^9 = \gamma^{+-}. \tag{A.5}$$

Other useful relations are

$$\gamma^{+-}\Pi\Pi' = 1, \quad (\gamma^{+-})^2 = \Pi^2 = (\Pi')^2 = 1, \tag{A.6}$$

$$\gamma^{+-}\gamma^\pm = \pm\gamma^\pm, \quad \bar{\gamma}^\pm\gamma^{+-} = \mp\bar{\gamma}^\pm, \quad \gamma^+\bar{\gamma}^+ = \gamma^-\bar{\gamma}^- = 0. \tag{A.7}$$

The 32-component spinors θ and Q of positive and negative chirality, respectively, are decomposed in terms of 16-component spinors as

$$\theta = \left(\begin{array}{c} \theta^\alpha \\ 0 \end{array} \right), \qquad Q = \left(\begin{array}{c} O \\ Q_\alpha \end{array} \right). \tag{A.8}$$

Abbreviated notations are used, in which the spinor indices are indicated by the positioning of matrices; frequently used expressions in the text include

$$\Omega Q \equiv \bar{\Omega}_\alpha{}^\beta Q_\beta; \quad Q\Omega \equiv Q_\alpha \Omega^\alpha{}_\beta; \quad \Omega^t Q \equiv (\bar{\Omega}^t)_\alpha{}^\beta Q_\beta; \quad Q\Omega^t \equiv Q_\alpha (\Omega^t)^\alpha{}_\beta; \tag{A.9}$$

$$\Omega^\alpha{}_\beta = (\gamma^{i_1})^{\alpha\gamma} .... (\bar{\gamma}^{i_q})_{\delta\beta}; \quad (\Omega^t)^\alpha{}_\beta = (\gamma^{i_q})^{\alpha\gamma} ... (\bar{\gamma}^{i_1})_{\delta\beta}; \tag{A.10}$$

$$\bar{\Omega}_\alpha{}^\beta = (\bar{\gamma}^{i_1})_{\alpha\gamma} .... (\gamma^{i_q})^{\delta\beta}; \quad (\bar{\Omega}^t)_\alpha{}^\beta = (\bar{\gamma}^{i_q})_{\alpha\gamma} .... (\gamma^{i_1})^{\delta\beta}. \tag{A.11}$$

In computing the algebras we need to use the following Fierz identity. For spinors θ_1 and θ_2 of the same chirality,

$$\begin{aligned} (\theta_1)^\alpha (\theta_2^\beta)^t \;\; = \;\; & -\frac{1}{16}(\theta_2 \bar{\gamma}^\mu \theta_1)(\gamma^\mu)^{\alpha\beta} + \frac{1}{96}(\theta_2 \bar{\gamma}_{\mu\nu\rho}\theta_1)(\gamma^{\mu\nu\rho})^{\alpha\beta} \\ & -\frac{1}{3840}(\theta_2 \bar{\gamma}_{\mu\nu\rho\sigma\tau}\theta_1)(\gamma^{\mu\nu\rho\sigma\tau})^{\alpha\beta}. \end{aligned} \tag{A.12}$$

Another useful identity is

$$\begin{aligned} M_{\alpha\beta} N_{\gamma\delta} \;\; = \;\; & -\frac{1}{32}\Big(2\gamma^\mu_{\alpha\delta}(N\bar{\gamma}_\mu M)_{\gamma\beta} - \frac{1}{3}\gamma^{\mu\nu\rho}_{\alpha\delta}(N\bar{\gamma}_{\mu\nu\rho} M)_{\gamma\beta} \\ & + \frac{1}{120}\gamma^{\mu\nu\rho\sigma\tau}_{\alpha\delta}(N\bar{\gamma}^{\mu\nu\rho\sigma\tau} M)_{\gamma\beta}\Big), \end{aligned} \tag{A.13}$$

with the understanding that free indices may be contracted with spinors of the same chirality.

B Symmetry currents and closed string superalgebra

To explicitly evaluate conserved charges we need the τ components of the symmetry currents in lightcone gauge. We review these here; we refer to [6] for the σ components. We also change the normalization of the generators as follows

$$P^- = \frac{P^{-\prime}}{p^+}, \quad J^{+I} = \sqrt{p^+} J^{+I\prime}, \quad P^I = \frac{P^{I\prime}}{\sqrt{p^+}}, \quad Q^{-\mathcal{I}} = \frac{Q^{-\mathcal{I}\prime}}{\sqrt{p^+}}. \tag{B.1}$$

The primed charges are the one we use in this paper, but from now we drop the primes. The reason for the rescaling is that with the new normalization the algebra depends on m rather than μ and it is the former that enters as a parameter in the lightcone action. Furthermore, the extension of the algebra that we discuss in section 3 is linear in P^+ and non-singular when $P^+ = 0$ only with the new normalizations. Finally, with the new normalization the energy of string states are properly of the order of the string mass, whereas with the previous normalization there was an extra factor of $1/p^+$.

The momenta are

$$\mathcal{P}^{+\tau} = p^+; \qquad \mathcal{P}^{I\tau} = \sqrt{p^+}(\cos(m\tau)\partial_\tau x^I + m\sin(m\tau)x^I). \tag{B.2}$$

The rotation currents are

$$\begin{aligned} \mathcal{J}^{+I\tau} &= \frac{1}{\sqrt{p^+}}(\mu^{-1}\sin(m\tau)\partial_\tau x^I - p^+ x^I\cos(m\tau)); \\ \mathcal{J}^{ij\tau} &= \left(x^i\partial_\tau x^j - x^j\partial_\tau x^i - \tfrac{1}{2}i(\theta^1\gamma^{-ij}\theta^1 + \theta^2\gamma^{-ij}\theta^2)\right), \end{aligned} \tag{B.3}$$

with a corresponding expression for $\mathcal{J}^{i'j'}$. The lightcone Hamiltonian is

$$\mathcal{P}^{-\tau} = p^+(\partial_\tau x^- + i(p^+)^{-1}(\theta^1\bar{\gamma}^-\partial_+\theta^1 + \theta^2\bar{\gamma}^-\partial_-\theta^2) - \mu m(x^I)^2 - 4i\mu\theta^1\bar{\gamma}^-\Pi\theta^2). \tag{B.4}$$

Using the Virasoro constraint, and the fermion field equations, we find that the onshell Hamiltonian is

$$H = -\mathcal{P}^{-\tau} = \frac{1}{2}\left((\partial_\tau x^I)^2 + (\partial_\sigma x^I)^2 + m^2(x^I)^2) + i(\theta^1\bar{\gamma}^-\partial_\tau\theta^1 + \theta^2\bar{\gamma}^-\partial_\tau\theta^2)\right). \tag{B.5}$$

Closed string kinematical and dynamical supercharge currents are

$$Q^{+1\tau} = 2\sqrt{p^+}\bar{\gamma}^-(\cos\mu x^+\theta^2 + \sin\mu x^+\Pi\theta^1); \tag{B.6}$$

$$Q^{+2\tau} = -2\sqrt{p^+}\bar{\gamma}^-(\cos\mu x^+\theta^1 - \sin\mu x^+\Pi\theta^2); \tag{B.7}$$

$$Q^{-1\tau} = 2(\partial_- x^I\bar{\gamma}^I\theta^1 - m x^I\bar{\gamma}^I\Pi\theta^2); \tag{B.8}$$

$$Q^{-2\tau} = 2(\partial_+ x^I\bar{\gamma}^I\theta^2 + m x^I\bar{\gamma}^I\Pi\theta^1). \tag{B.9}$$

Closed string worldsheet symmetries are

$$P_n^{1I\tau} = (\partial_\tau x^I\tilde{\phi}_{-n} - x^I\partial_\tau\tilde{\phi}_{-n}); \tag{B.10}$$

$$P_n^{2I\tau} = (\partial_\tau x^I\phi_{-n} - x^I\partial_\tau\phi_{-n}); \tag{B.11}$$

$$Q_{-n}^{1\tau} = 2\bar{\gamma}^-(\theta^1 - i d_n\Pi\theta^2)c_n\tilde{\phi}_n; \tag{B.12}$$

$$Q_{-n}^{2\tau} = 2\bar{\gamma}^-(\theta^2 + i d_n\Pi\theta^1)c_n\phi_n, \tag{B.13}$$

where

$$\phi_n(\tau,\sigma) = e^{-i(w_n\tau + n\sigma)}, \qquad \tilde{\phi}_n(\tau,\sigma) = e^{-i(w_n\tau - n\sigma)}. \tag{B.14}$$

Open string symmetries are appropriate combinations of these as discussed in [6] and in the main text. In particular, the bosonic symmetries in the Neumann and Dirichlet directions, labelled by r and r' respectively are

$$\sqrt{|\omega_n|}P_n^{r\tau} = (\partial_\tau x^r f_n - x^r \partial_\tau f_n); \tag{B.15}$$

$$\sqrt{|\omega_n|}P_n^{r'\tau} = (\partial_\tau (x^{r'} - x_0^{r'}(\tau,\sigma))\tilde{f}_n - (x^{r'} - x_0^{r'}(\tau,\sigma))\partial_\tau \tilde{f}_n), \tag{B.16}$$

where $f_n = e^{-i\omega_n \tau}\cos(n\sigma)$ and $\tilde{f}_n = -ie^{-i\omega_n\tau}\sin(n\sigma)$. The Dirichlet zero modes $x_0^{r'}(\tau,\sigma)$ are as given in (2.10) and (2.11).

The special kinematical supercharge symmetry current for the D_+ branes is given by

$$\begin{aligned} \hat{Q}^{+\tau} &= 2\sqrt{p^+}\bar{\gamma}^- \left(\sqrt{\frac{\pi m}{2(e^{2m\pi}-1)}} e^{m\sigma}(\mathcal{P}_+\theta^1 + \Pi\mathcal{P}_+\theta^2) \right. \\ &\left. + \sqrt{\frac{\pi m}{2(1-e^{-2m\pi})}} e^{-m\sigma}(\mathcal{P}_-\theta^1 - \Pi\mathcal{P}_-\theta^2) \right). \end{aligned} \tag{B.17}$$

The symmetry superalgebra of the pp-wave background is as follows. The commutators of the bosonic generators are[5]

$$\begin{aligned} [P^-, P^I] &= im^2 J^{+I}, \quad [P^I, J^{+J}] = i\delta^{IJ}P^+, \quad [P^-, J^{+I}] = -iP^I, \\ \left[P^i, J^{jk}\right] &= -i(\delta^{ij}P^k - \delta^{ik}P^j), \quad [P^{i'}, J^{j'k'}] = -i(\delta^{i'j'}P^{k'} - \delta^{i'k'}P^{j'}), \\ \left[J^{+i}, J^{jk}\right] &= -i(\delta^{ij}J^{+k} - \delta^{ik}J^{+j}), \quad [J^{+i'}, J^{j'k'}] = -i(\delta^{i'j'}J^{+k'} - \delta^{i'k'}J^{+j'}), \\ \left[J^{ij}, J^{kl}\right] &= -i(\delta^{jk}J^{il} + ...), \quad [J^{i'j'}, J^{k'l'}] = -i(\delta^{j'k'}J^{i'l'} + ...), \end{aligned} \tag{B.18}$$

where the ellipses denote permutations, whilst the commutation relations between the even and odd generators are

$$\begin{aligned} \left[J^{ij}, Q^\pm\right] &= -\tfrac{1}{2}iQ^\pm(\gamma^{ij}), \quad [J^{i'j'}, Q^\pm] = -\tfrac{1}{2}iQ^\pm(\gamma^{i'j'}), \\ \left[J^{+I}, Q^-\right] &= -\tfrac{1}{2}iQ^+(\gamma^{+I}), \\ \left[P^I, Q^-\right] &= \tfrac{1}{2}mQ^+(\Pi\gamma^{+I}), \quad [P^-, Q^+] = mQ^+\Pi, \end{aligned} \tag{B.19}$$

and the anticommutation relations are

$$\begin{aligned} \{Q^+, \bar{Q}^+\} &= 2P^+\bar{\gamma}^-, \\ \{Q^+, \bar{Q}^-\} &= (\bar{\gamma}^-\gamma^+\bar{\gamma}^I)P^I - im(\bar{\gamma}^-\gamma^+\bar{\gamma}^I\Pi)J^{+I}, \\ \{Q^-, \bar{Q}^+\} &= (\bar{\gamma}^+\gamma^-\bar{\gamma}^I)P^I - im(\bar{\gamma}^+\gamma^-\bar{\gamma}^I\Pi)J^{+I}, \\ \{Q^-, \bar{Q}^-\} &= 2\bar{\gamma}^+P^- + im(\gamma^{+ij}\Pi)J^{ij} + im(\gamma^{+i'j'}\Pi')J^{i'j'}. \end{aligned} \tag{B.20}$$

It is useful to give the latter in terms of the real supercharges $Q^+ = (-Q^{+2} + iQ^{+1})/\sqrt{2}$ and $Q^- = (Q^{-1} + iQ^{-2})/\sqrt{2}$:

$$\{Q^{+1}, Q^{+1}\} = \{Q^{+2}, Q^{+2}\} = 2\bar{\gamma}^-P^+; \quad \{Q^{+1}, Q^{+2}\} = \{Q^{+2}, Q^{+1}\} = 0;$$

[5] Notice our rotational generators differ from the ones in [4] by a factor of i.

$$
\begin{aligned}
\{Q^{+1}, Q^{-2}\} &= -\{Q^{+2}, Q^{-1}\} = \bar\gamma^{-}\gamma^{+}\bar\gamma^{I} P^{I}; \\
\{Q^{+1}, Q^{-1}\} &= \{Q^{+2}, Q^{-2}\} = -m\bar\gamma^{-}\gamma^{+}\bar\gamma^{I}\Pi J^{+I}; \\
\{Q^{-2}, Q^{+1}\} &= -\{Q^{-1}, Q^{+2}\} = \bar\gamma^{+}\gamma^{-}\bar\gamma^{I} P^{I}; \\
\{Q^{-1}, Q^{+1}\} &= \{Q^{-2}, Q^{+2}\} = m\bar\gamma^{+}\gamma^{-}\bar\gamma^{I}\Pi J^{+I}; \\
\{Q^{-1}, Q^{-1}\} &= \{Q^{-2}, Q^{-2}\} = 2\bar\gamma^{+} P^{-}; \\
\{Q^{-2}, Q^{-1}\} &= -\{Q^{-1}, Q^{-2}\} = m(\gamma^{+ij}\Pi J^{ij} + \gamma^{+i'j'}\Pi' J^{i'j'}).
\end{aligned} \tag{B.21}
$$

C Mode expansions for closed strings

In our conventions the closed string mode expansions are given by

$$
x^{I}(\sigma,\tau) = \cos(m\tau)x_0^{I} + m^{-1}\sin(m\tau)p_0^{I} + i\sum_{n\neq 0}\omega_n^{-1}(\alpha_n^{1I}\tilde\phi_n + \alpha_n^{2I}\phi_n); \tag{C.1}
$$

$$
\theta^{1}(\sigma,\tau) = \theta_0^{1}\cos(m\tau) + \Pi\theta_0^{2}\sin(m\tau) + \sum_{n\neq 0} c_n\left(i d_n\Pi\theta_n^{2}\phi_n + \theta_n^{1}\tilde\phi_n\right); \tag{C.2}
$$

$$
\theta^{2}(\sigma,\tau) = \theta_0^{2}\cos(m\tau) - \Pi\theta_0^{1}\sin(m\tau) + \sum_{n\neq 0} c_n\left(-i d_n\Pi\theta_n^{1}\tilde\phi_n + \theta_n^{2}\phi_n\right), \tag{C.3}
$$

where the expansion functions are given in (B.14). After canonical quantization we get the following (anti)commutators

$$
[p_0^{I}, x_0^{J}] = -i\delta^{IJ}, \qquad [\alpha_m^{\mathcal{I}I}, \alpha_n^{\mathcal{J}J}] = \tfrac{1}{2}\omega_m\delta_{n+m,0}\delta^{\mathcal{I}\mathcal{J}}\delta^{IJ}, \tag{C.4}
$$

$$
\{\theta_0^{\mathcal{I}}, \theta_0^{\mathcal{J}}\} = \frac{1}{4}(\gamma^{+})\delta^{\mathcal{I}\mathcal{J}}, \qquad \{\theta_m^{\mathcal{I}}, \theta_n^{\mathcal{J}}\} = \frac{1}{4}(\gamma^{+})\delta^{\mathcal{I}\mathcal{J}}\delta_{m+n,0}, \tag{C.5}
$$

where $\mathcal{I} = 1, 2$. It is convenient to introduce creation and annihilation operators

$$
a_0^{I} = \frac{1}{\sqrt{2m}}(p_0^{I} + imx_0^{I}), \qquad \bar a_0^{I} = \frac{1}{\sqrt{2m}}(p_0^{I} - imx_0^{I}), \qquad [\bar a_0^{I}, a_0^{J}] = \delta^{IJ}. \tag{C.6}
$$

Expressed in terms of these modes the spacetime charges are

$$
P^{+} = p^{+}, \qquad P^{I} = \sqrt{p^{+}}p_0^{I}, \qquad J^{+I} = -x_0^{I}\sqrt{p^{+}}, \tag{C.7}
$$

$$
Q^{+1} = 2\sqrt{p^{+}}\bar\gamma^{-}\theta_0^{2}, \qquad Q^{+2} = -2\sqrt{p^{+}}\bar\gamma^{-}\theta_0^{1}. \tag{C.8}
$$

Note that the complex Q^{+} appearing in the closed string algebra is

$$
Q^{+} = \frac{1}{\sqrt{2}}(iQ^{+1} - Q^{+2}) = 2\sqrt{p^{+}}\bar\gamma^{-}\theta_0. \tag{C.9}
$$

The rotation charges are

$$
\begin{aligned}
J^{IJ} &= -i(a_0^{I}\bar a_0^{J} - a_0^{J}\bar a_0^{I} + \tfrac{1}{2}\sum_{\mathcal{I}=1,2}\theta_0^{\mathcal{I}}\gamma^{-IJ}\theta_0^{\mathcal{I}}) \\
&\quad -i\sum_{\mathcal{I}=1,2}\sum_{n>0}\left(2\omega_n^{-1}(\alpha_{-n}^{\mathcal{I}I}\alpha_n^{\mathcal{I}J} - \alpha_{-n}^{\mathcal{I}J}\alpha_n^{\mathcal{I}I}) + \theta_{-n}^{\mathcal{I}}\gamma^{-IJ}\theta_n^{\mathcal{I}}\right).
\end{aligned} \tag{C.10}
$$

The Hamiltonian is

$$
\begin{aligned}
H &= \frac{1}{2}(p_0^2 + m^2 x_0^2) + 2im(\theta_0^1 \bar{\gamma}^- \Pi \theta_0^2) \\
&\quad + \sum_{\mathcal{I}=1,2} \sum_{n \neq 0} (\alpha_{-n}^{\mathcal{I}I} \alpha_n^{\mathcal{I}I} + \omega_n \theta_{-n}^{\mathcal{I}} \bar{\gamma}^- \theta_n^{\mathcal{I}}).
\end{aligned}
\tag{C.11}
$$

Finally the dynamical supercharges take the form

$$
\begin{aligned}
Q^{-1} &= \Big((2p_0^I \bar{\gamma}^I \theta_0^1 - 2m x_0^I \bar{\gamma}^I \Pi \theta_0^2) \\
&\quad + \sum_{n>0} \left(4c_n \alpha_{-n}^{I1} \bar{\gamma}^I \theta_n^1 + \frac{2im}{\omega_n c_n} \alpha_{-n}^{2I} \bar{\gamma}^I \Pi \theta_n^2 + h.c. \right) \Big) \\
Q^{-2} &= \Big((2p_0^I \bar{\gamma}^I \theta_0^2 + 2m x_0^I \bar{\gamma}^I \Pi \theta_0^1) \\
&\quad + \sum_{n>0} \left(4c_n \alpha_{-n}^{I2} \bar{\gamma}^I \theta_n^2 - \frac{2im}{\omega_n c_n} \alpha_{-n}^{1I} \bar{\gamma}^I \Pi \theta_n^1 + h.c. \right) \Big)
\end{aligned}
\tag{C.12}
$$

The complex Q^- appearing in the algebra is $(Q^{-1} + iQ^{-2})/\sqrt{2}$.

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